Tidal prediction - OXP2003 Lecture 5

- Practical methods of tide prediction are based on Laplace's principle (1775) which states that, for every constituent of the equilibrium tide there exits a harmonic constituent in the real tide with the same frequency but with different amplitude and phase (e.g. the M_2 constituent may be bigger, high water may occur later than the equilibrium tide, but the period is still 12.42 hours).
- Any tidal curve is the sum of a large number of these constituents. This method was first used practically by Sir George Darwin (Charles' nephew) around 1880.
- Predictions for standard ports typically use the sum of 60 constituents (the "standard harmonic method" due to Doodson, 1921) but to accurately describe the tide in many estuaries involves the summation of 100 constituents (the "extended harmonic method").
- Reasonably accurate predictions can be made just using M_2, S_2, K_1 and O_1
- Try this on a spreadsheet using the following angular frequencies
  - M_2 $2\pi / 12.42 = 0.5059$ radians/hour
  - S_2 $2\pi / 12.00 = 0.5236$ radians/hour
  - O_1 $2\pi / 26.87 = 0.2338$ radians/hour

The harmonic method of prediction

- Mathematically, the tidal elevation at any place can be written as below. (It is conventional to use the Greek letter zeta, ζ, for tides. Sorry….η for waves but ζ for tides.)

$$\zeta = Z_0 + \sum^{N} H_n \cos(\omega_n t - \alpha_n - g_n)$$

- N refers to the sum of all constituents, H_n is the amplitude of the n^{th} constituent, ω_n is the angular speed of that constituent, α_n is its phase relative to lunar transit of the equilibrium tide and g_n is the phase lag of the real tide to the equilibrium tide.
- The term Z_0 is the difference between mean sea level and chart datum (approximately the level of Lowest Astronomical Tide, or LAT). This ensures that the predicted value of ζ is always positive. This is preferred by mariners.
- A further correction is required for the 18.61 yearly regression of the lunar nodes. The nodal factors h and ε on the next slide complete the practical equation that is used for tide prediction worldwide.

Harmonic analysis

$$\zeta = Z_0 + \sum h_n H_n \cos(\omega_n t - \alpha_n - g_n + \varepsilon_n)$$

- In the above equation, all the frequencies and constants relating to the equilibrium tide are known. All that is required to make prediction possible is analysis of the values of H, g and Z_0, at every location where they are required.
- This is done through observations of tidal elevation using accurate tide gauges (e.g. stilling wells or pressure recording tide gauges such as pneumatic gauges).
- The length of record required depends on the number of constituents that need to be identified. A 29 day analysis allows 14 astronomical constituents to be deduced (Doodson, 1954). Over half of the standard ports in the UK use a full year of data in their tidal analyses.
- Fourier analysis is a mathematical technique for deducing the amplitudes and phases of tidal constituents from observations. But, an uninterrupted time series at regular intervals is needed.
- Least squares fitting is more forgiving and allows data gaps and shorter time series.

Least squares tidal analysis

- To illustrate the method we will consider just the M_2 constituent. The theoretical tide (ignoring nodal corrections - that would require a 19 year time series) in this case is given by:

$$\zeta_{M2} = Z_0 + H_{M2} \cos(\omega_{M2} t - g_{M2})$$

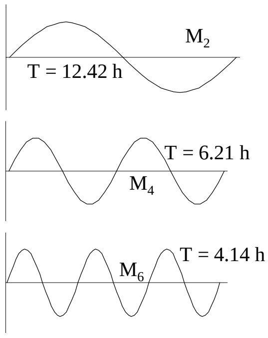
- Least squares fitting subtracts the theoretical value from the observed value, at every point where a reading is available and then minimises the sum of the squares. It can be thought of as a multiple regression approach.
- Using a series of observations, z(t), the difference between the observed tide and the predicted tide is $(z - \zeta_{M2})$.
- The sum of squared error $S = \sum \left(z(t) - \zeta(t)\right)^2$
- . The value of S is then minimised to obtain Z_0, H_{M2} and g_{M2}

Shallow water constituents

- Making tidal predictions on the basis of frequencies present in the equilibrium tide does not allow for
  - meteorological perturbations
  - shallow water constituents
- When the tidal wave enters shallow water (less than 50 m), several physical processes act to modify and distort the shape of the waveform. Typically, the time between low water and high water shortens giving the tidal curve a sharp rise and a slower fall. The processes responsible are:
  - Bottom friction
  - Non-linearity in the equations
  - The wave amplitude is no longer small compared to the depth
- The result is that higher harmonics appear in the equations. These have frequencies that are simple multiples of the fundamental frequencies. The new shallow water terms are sometimes called "overtides". They are generated in shelf seas, and do not exist in the equilibrium tide.
- These shallow water constituents must be included for accurate tidal predictions in shelf seas and estuaries.

Shallow water constituents

- Shallow water distortion of the M_2 constituent gives rise to new constituents M_4 (twice the M_2 frequency), M_6 (three times), etc.
- In certain areas the shallow water effects take the form of double high water (Southampton) or double low water (Portland, Hook of Holland).
- The double HW at Southampton is NOT caused by two separate waves being split by the Isle of Wight. It is a combination of $M_2 + M_4 + M_6$ constituents.
- For these effects to be significant the amplitude of M_4 needs to be large compared to M_2. For double high waters, the ratio of M_4 to M_2 amplitude needs to be greater than 0.25. This is the case around southern England where there is a degenerate M_2 amphidrome.



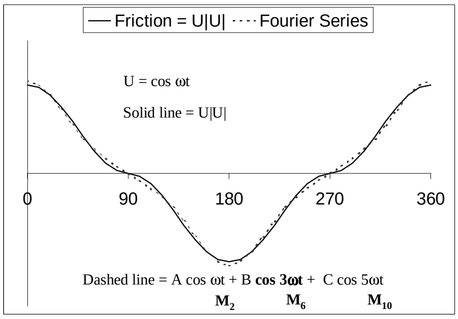
Generating the shallow water constituents

- The M_4 shallow water constituent is usually the largest of the shallow water constituents. This originates through non-linearities in the governing equations (which is when one of the variables is multiplied by another).
- An example is the change in water column height that arises in the continuity equation

$$\frac{1}{(Z_0+\zeta)}\frac{\partial \zeta}{\partial t}$$

- Terms like M_4 that oscillate with twice the semidiurnal frequency are called quarter diurnal, or (sometimes) species 4
- It can be shown that the quadratic drag law for friction at the bed , $-kU^2$ (as in the first practical) gives rise to harmonics of species 6 and 10. (i.e. six cycles per day and ten cycles per day).

M_6 species from the friction term



Shallow water interactions

- Interaction between constituents can also occur (called "intermodulation"). For example, interaction between M_2 and S_2 gives rise to a shallow water constituent MS_4 whose frequency is the sum of the original frequencies (0.5059 + 0.5236 rad h^{-1}).
- Non-linear interactions between the tidal constituents (M_2, S_2, O_1, K_1, etc) can generate a whole range of shallow water terms.
- In a semidiurnal regime, only the even harmonics are important (M_2 M_4 M_6, etc) but in a mixed regime there can be interactions between diurnal and semidiurnal tides giving rise to odd harmonics (e.g. in Anchorage, Alaska, species 5 tides are observed).
- To appreciate the relative size of shallow water constituents, at Liverpool:
  - M_2 305 cm
  - M_4 22 cm
  - M_6 6 cm
  - M_8 3 cm
  - MS_4 12 cm
- The magnitude of the M_4 tide is proportional to the square of the magnitude of the M_2 tide ($M_4 \propto M_2^2$). Shallow water distortion is therefore most severe at spring tides.

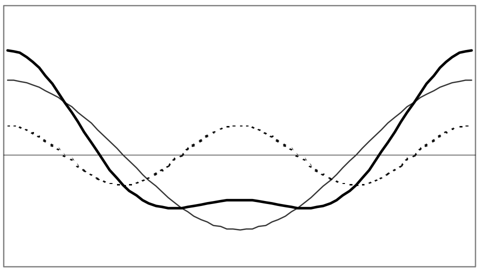
Double high waters (low waters)

- Tidal predictions for most UK ports can be made fairly accurately by including shallow water constituents up to species 6.
- Tides become more severely distorted in estuaries. Prediction requires a more complex treatment (100 terms in the extended harmonic method) with the inclusion of shallow water constituents up to species 10.
- A qualitative picture of the double high and low waters that occur at certain ports (e.g. Southampton and Portland) can be obtained by constructing a tide comprising just the M_2 and M_4 constituents.

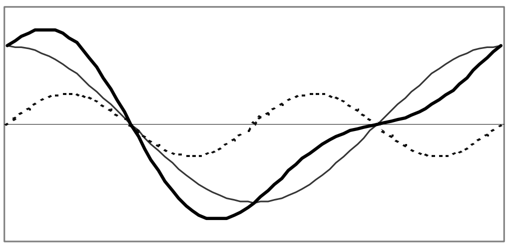
$$\zeta = \underbrace{A_2 \cos \omega t}_{\mathbf{M_2}} + \underbrace{A_4 \cos (2\omega t - \lambda)}_{\mathbf{M_4}}$$

- In the following graphs the M_4 amplitude (A_4) is taken as 0.4 times the M_2 amplitude (A_2). Different tidal curves result (the bold lines), depending on the relative phase, λ , of M_4 to M_2.
- NB. For the double high or low waters to occur, $A_4/A_2 \geq 0.25$

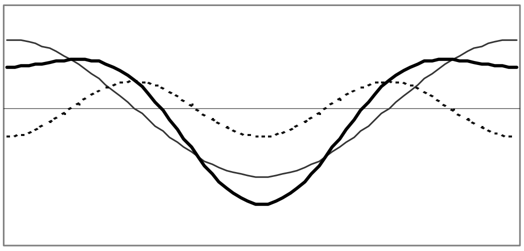
Double low water (λ = 0)



Slow rise, rapid fall (λ = 90)



Double high water (λ = 180)



Fast rise, slow fall (λ = 270)