Hukum NEWTON II

$F = m \times a$ dgn F : gaya
m : massa
a : percepatan

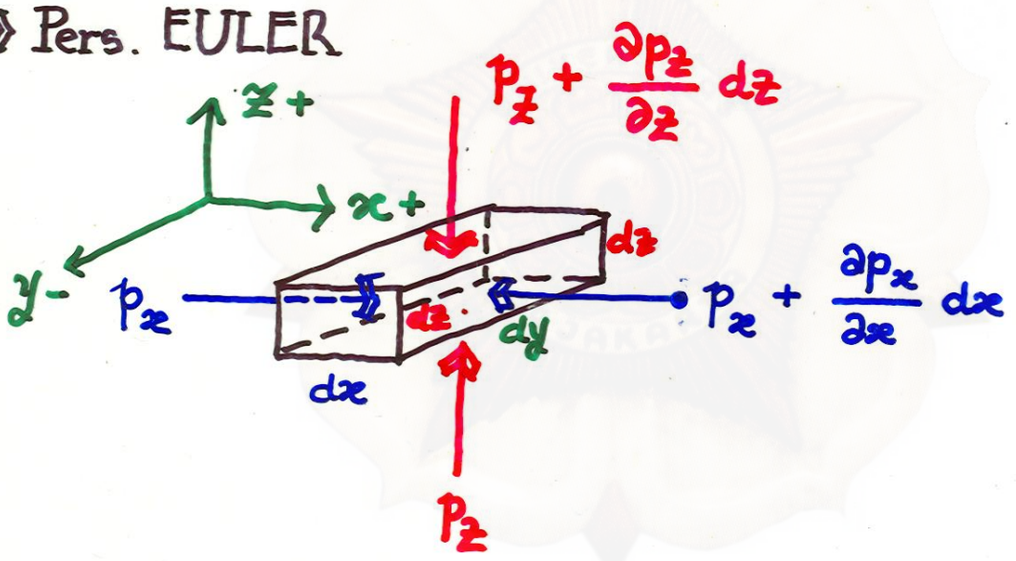
Definisi : S.I. $1\ N(ewton) = 1\ kg_m \times 1\ m/det^2$

M.K.S. $1\ kg_{force} = 1\ kg_m \times g\ m/det^2$

$F = m \times a$

Jadi $\boxed{1\ kg_f = g\ N(ewton)}$

Pers. EULER



Gaya arah x :

$$\underbrace{p_x}_{\text{tek.}} \underbrace{dy\,dz}_{\text{luas}} - \underbrace{\left(p_x + \frac{\partial p_x}{\partial x} dx\right)}_{\text{tekanan}} \underbrace{dy\,dz}_{\text{luas}} + \underbrace{\rho\, dx\, dy\, dz}_{\text{massa}} \cdot \underbrace{a_x}_{\text{(percepatan)}} = 0$$

Jadi $\frac{\partial p_x}{\partial x} = \rho a_x$ gaya per satuan massa

Analogi : $\frac{\partial p_y}{\partial y} = \rho a_y$ dan $\frac{\partial p_z}{\partial z} = \rho a_z$

Secara umum :

$$p = p(x, y, z)$$

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$$

atau

$$\boxed{dp = \rho a_x dx + \rho a_y dy + \rho a_z dz}$$ Pers. EULER

- **Bidang equipotensial**

  yaitu suatu bidang tempat kedudukan titik-titik yang bertekanan hidrostatis sama.

Syarat bidang equipotensial :

$$p = C \text{ (konstanta)}$$

$$dp = 0$$

$$\boxed{a_x dx + a_y dy + a_z dz = 0}$$ Pers. Bid. Eq. Pot. (PBEP)

Contoh :

a. Zat cair diam :

$$a_x = 0, \quad a_y = 0, \quad a_z = -g$$

PBEP : $-g\,dz = \phi$

$$dz = 0$$

$$\int dz = C \rightarrow \boxed{z = \text{Constan}}$$

Jadi dlm zat cair diam, PBEP merupakan bidang horisontal.

Besarnya tekanan dihitung sbb :

Pers. EULER :

$$dp = -\rho g\,dz \rightarrow \boxed{p = -\rho g z + C}$$

c. Zat cair dlm tangki dipercepat vertikal.

PBEP : $(-g + a_z)\, dz = 0$

z = Constan bid. horisontal

Tekanan : $dp = (-g + a_z)\, \rho\, dz$

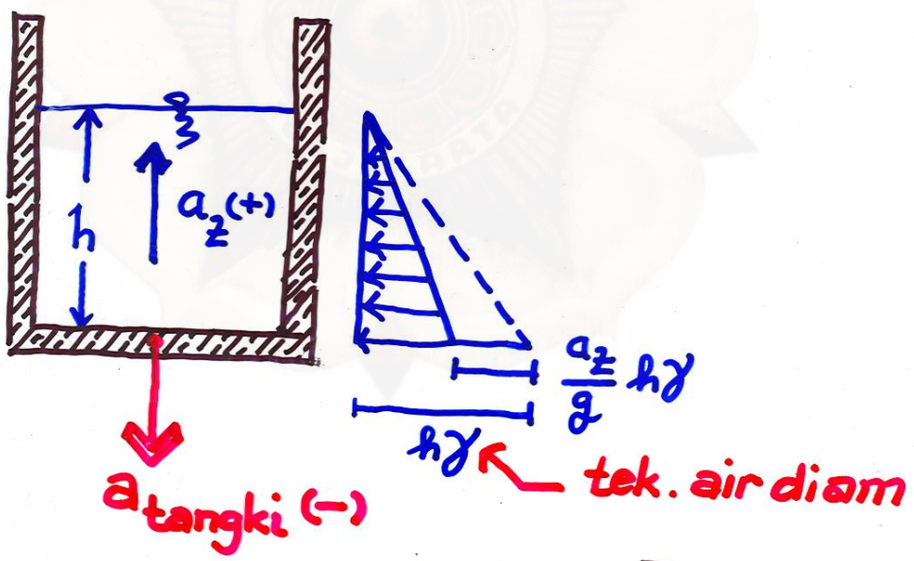
$$p = (-g + a_z)\, \rho z + C$$

$$p = \left(1 - \frac{a_z}{g}\right) \gamma z + C$$

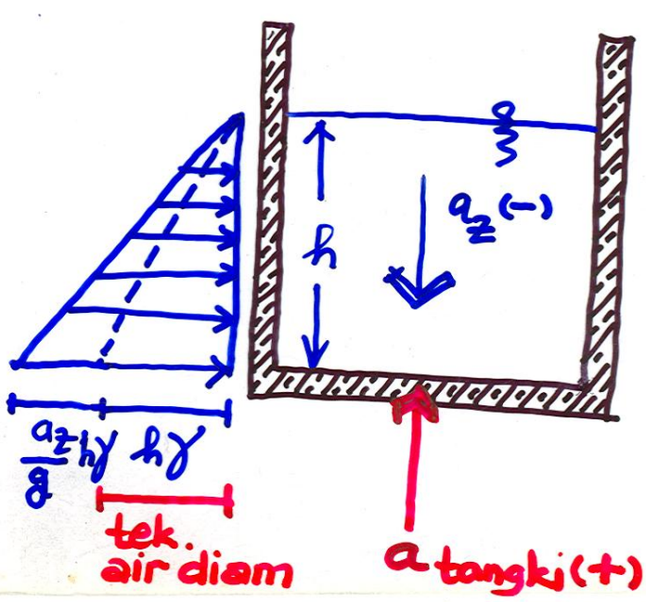
dengan a_z adalah percepatan yg dialami cairan

+ arah keatas - arah kebwh.

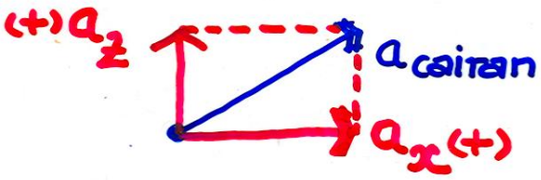
Percepatan yg dialami cairan selalu berlawanan dengan percepatan tangkinya.



$$a_z = -a_{tangki}$$



d. Zat cair dlm tangki yg dipercepat miring



PBEP :

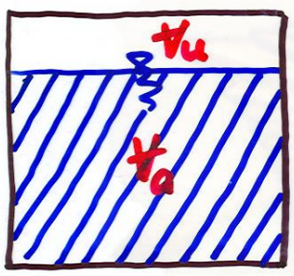
$$a_x dx + (a_z - g) dz = 0$$

$$z = \frac{-a_x}{a_z - g} x + C$$

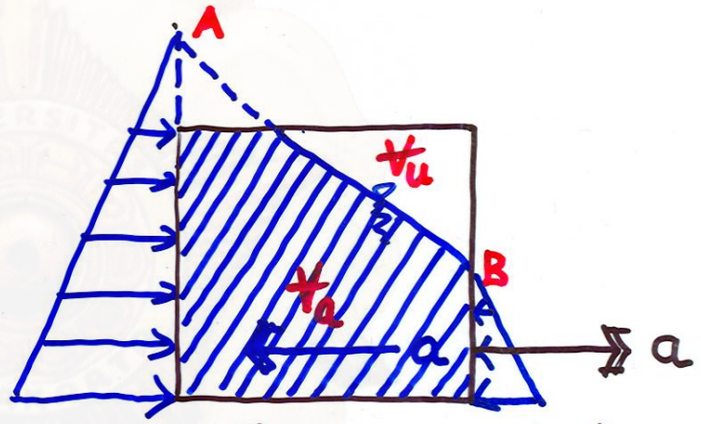
Catatan : $a_{cairan} = -a_{tangki}$

e. Zat cair dlm tangki tertutup dipercepat

- Berisi sebagian



1. Keadaan diam



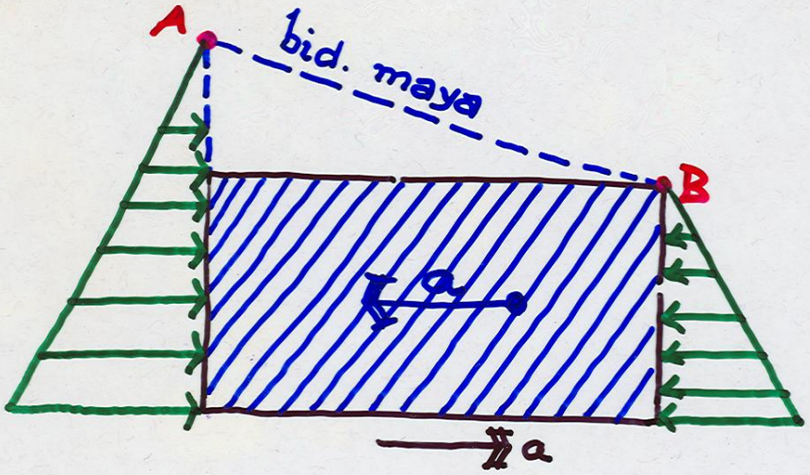
2. Tangki dipercepat kekanan

Dasar perhitungan :

1. Kemiringan grs AB sudah tertentu, ditentukan oleh PBEP.
2. Volume air dan udara pada Kondisi 1 dan Kondisi 2 adalah sama

∴ Arah grs AB : $\alpha = \arctan\left(\frac{-a}{g}\right)$

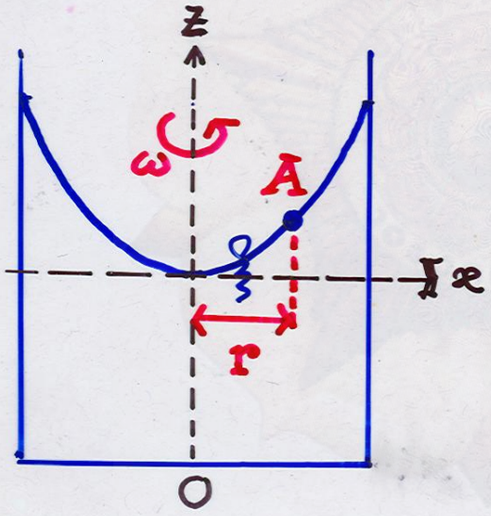
- Berisi penuh



Dasar perhitungan : arah grs AB

$$\alpha = \text{arc.tan}\left(-\frac{a}{g}\right)$$

f. Zat cair didalam silinder berotasi



Jika suatu silinder diputar dengan kecepatan sudut ω radian/det, maka setiap titik didalam air akan mengalami gaya sentrifugal (arah keluar) sebesar

$$F = m.\omega^2 r$$

Catatan :

Jika silinder berputar N rpm (rotation per menit) maka

$$\omega = \frac{N \times 2\pi}{60} \quad \frac{\text{rad}}{\text{det}}$$

PBEP :

$$a_x\,dx + a_z\,dz = 0$$

$$\omega^2 x\,dx - g\,dz = 0$$

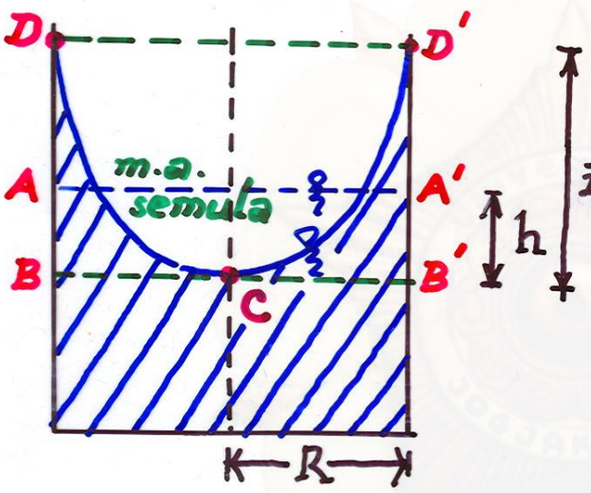
atau $z = \frac{\omega^2}{2g} x^2 + C$ ← parabola

Dipilih ttk terendah pada muka air sbg salib sumbu yaitu ttk O. → $x = 0, z = 0$, shg $C = \phi$.

Pers. Bidang Equipotensial menjadi:

$$\boxed{z = \frac{\omega^2}{2g} x^2}$$ merupakan paraboloida

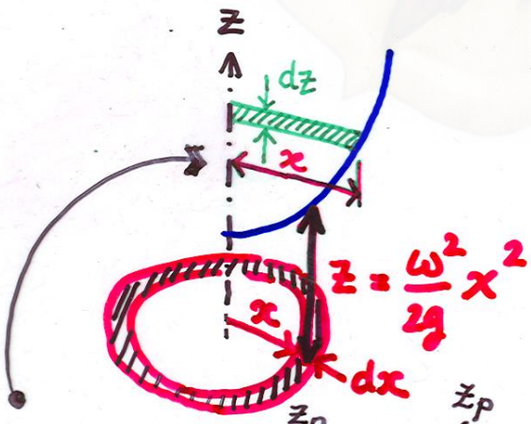
Beberapa fakta mengenai paraboloida PBEP:



» $z_p = \frac{\omega^2 R^2}{2g}$

» Volume air sesudah dan sebelum rotasi adalah sama, shg

$\forall_{par.\ DBB'D'CD} = \forall_{sil.\ ABB'A'}$



» $\forall_{par.\ udara} = \int_0^{z_p} \pi x^2 dz = \int_0^{z_p} \pi \frac{2gz}{\omega^2} dz$

» Vol. paraboloida DBB'D'CD:

$$\forall = \int_{x=0}^{x=R} \underbrace{2\pi x \cdot dx}_{A\ arsir} \cdot z$$

$$= \frac{2\pi\omega^2}{2g} \int_0^R x^3 dx$$

$$\forall_p = \frac{1}{4} \pi \frac{\omega^2 R^4}{g}$$

» Vol. silinder DBB'D':

$$\forall_s = \pi R^2 z_p$$

$$= \pi R^2 \frac{\omega^2 R^2}{2g} = \frac{1}{2} \pi \frac{\omega^2 R^4}{g}$$

$$\boxed{\forall_p = \frac{1}{2} \forall_s}$$

o) Volume silinder ABB'A' = Volume paraboloida DBB'D'CD

$$\pi R^2 h = \frac{1}{4}\pi \frac{\omega^2 R^4}{g}$$

$$\therefore \boxed{h = \frac{1}{4}\frac{\omega^2 R^2}{g}} \qquad \boxed{z_p = \frac{1}{2}\frac{\omega^2 R^2}{g}}$$

$$\boxed{h = \frac{1}{2} z_p}$$

Jadi elevasi muka air semula berada di tengah-tengah el.m.a. terendah dan tertinggi.

RESUME

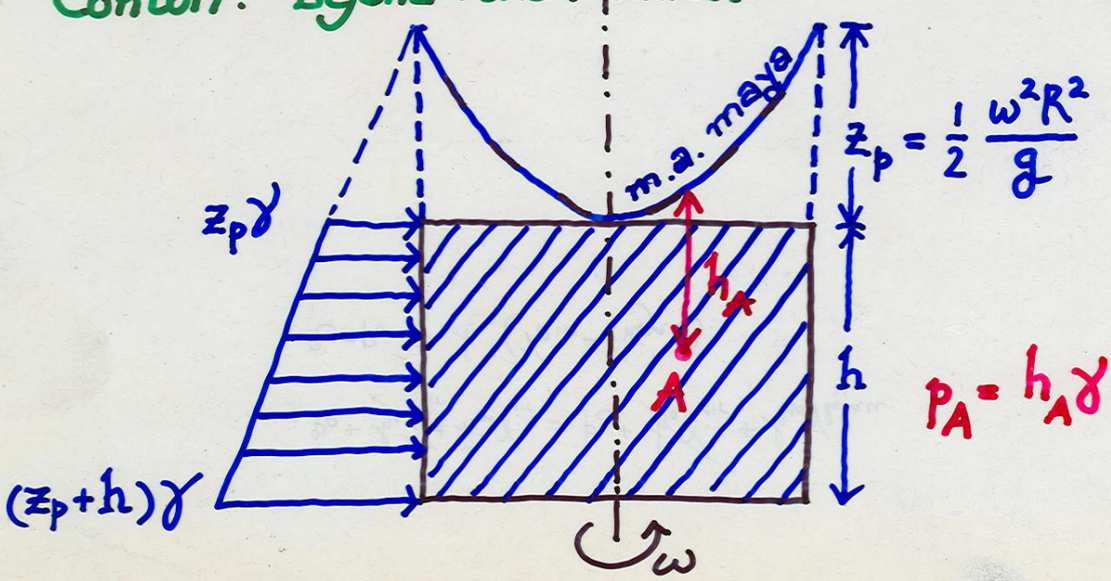
1. $h = \frac{1}{2} z_p$

2. $z_p = \frac{1}{2}\frac{\omega^2 R^2}{g}$

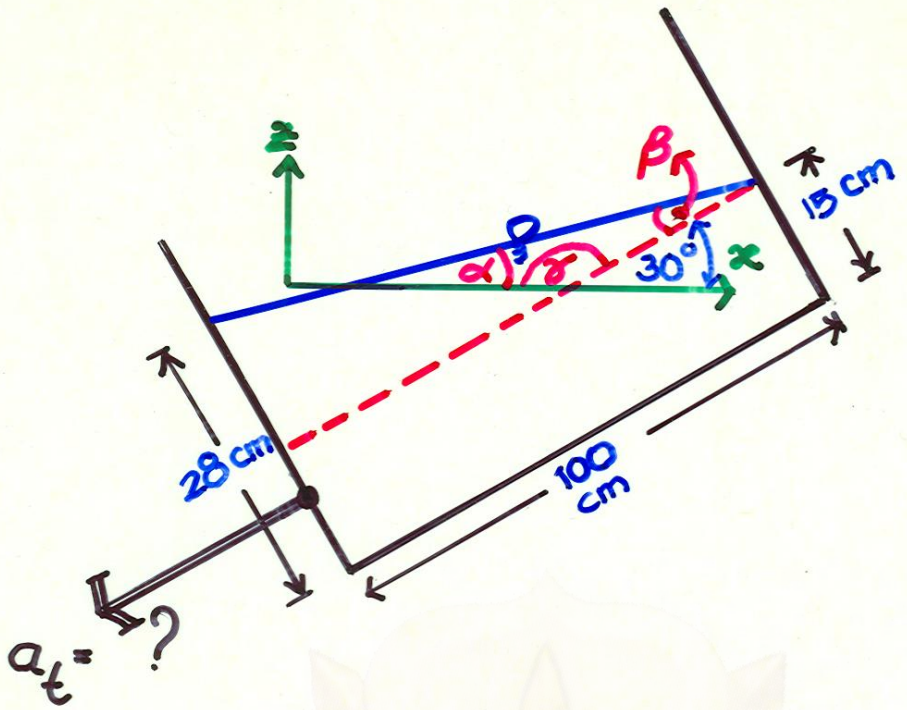
3. Jika tidak ada yg tumpah volume air tetap sama

4. Volume paraboloida (udara) yg terjadi = $\frac{1}{2}$ volume silinder yg menyelimuti.

Contoh: Bejana Penuh Cairan



5.5.2



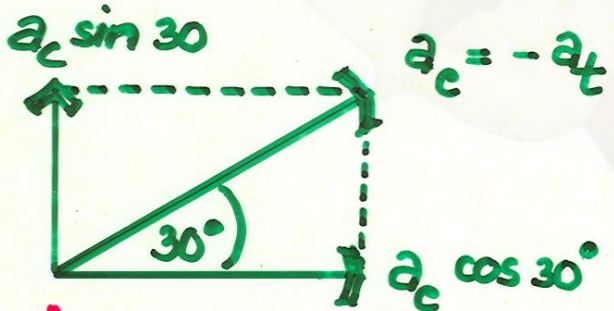
• Dari gambar tampak :

$$\tan\beta = \frac{28-15}{100} = 0.13 \rightarrow \beta = 7.407^\circ$$

$$\gamma = 180^\circ - 30^\circ = 150^\circ$$

sehingga $\alpha = 180 - \beta - \gamma = 22.593^\circ \rightarrow \tan\alpha = 0.416$

• Percepatan cairan :



$g = 9.81\ m/d^2$

• PBEP :

$$a_x dx + a_y dy + a_z dz = 0 \quad (a_y = 0)$$

$$z = -\frac{a_x}{a_z} x + C$$

Arah grs : $\tan\alpha = -\frac{a_x}{a_z}$

$$0.416 = -\frac{a_c \cos 30^\circ}{a_c \sin 30^\circ - 9.81}$$

$$a_c = +3.8\ m/det^2$$

Jadi percepatan tangki :

$$a_t = -a_c = -3.8\ m/det^2$$

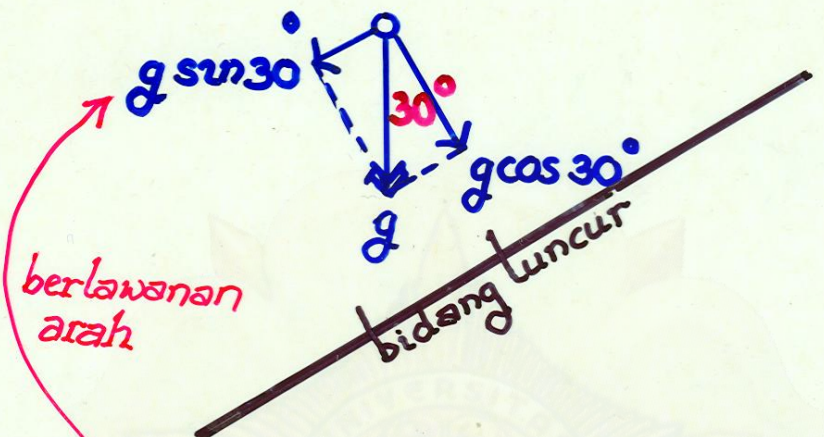
S.5.3

Soal:

Sebuah tangki berisi air meluncur tanpa gesekan pd bid. miring 30° thd horisontal. Bgmn kedudukan air dlm tangki?

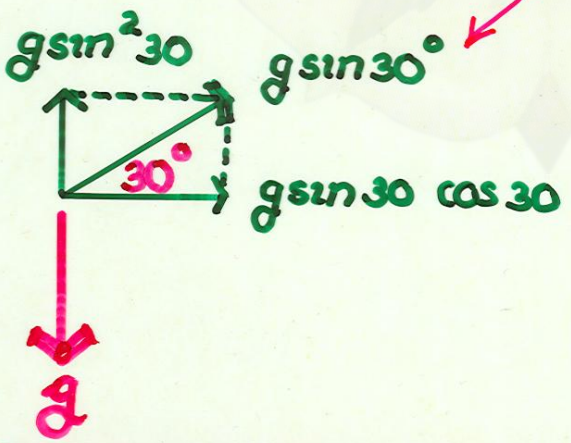
Jawab:

● Tangki meluncur tanpa gesekan, maka percepatannya :



1) Komponen $g \cos 30^\circ$ ditahan oleh bidang luncur
2) Komponen $g \sin 30^\circ$ akan menyebabkan tanki meluncur

● Percepatan yg dialami cairan



● Kemiringan muka air : $\tan\alpha = -\frac{a_x}{a_z} = -\frac{g \sin 30^\circ \cos 30^\circ}{g \sin^2 30^\circ - g}$

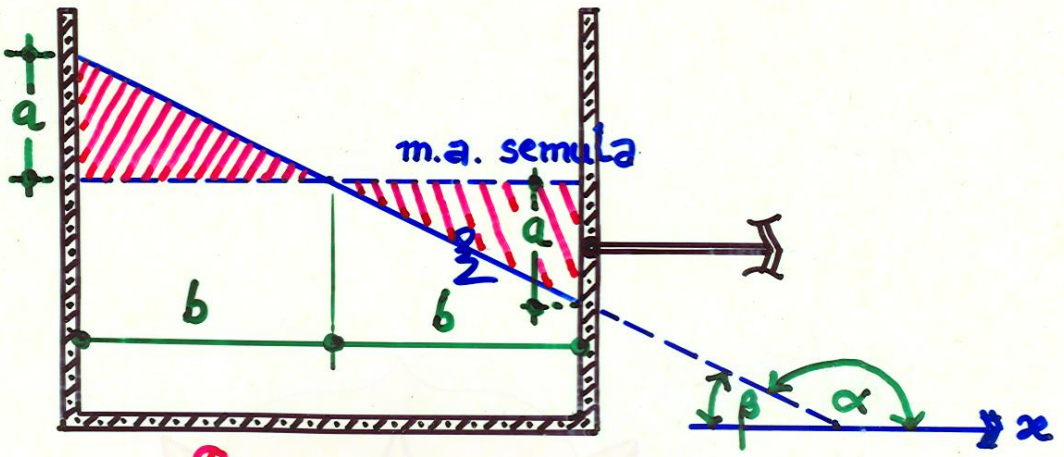
$\therefore \tan\alpha = +\frac{\sin 30^\circ \cos 30^\circ}{\cos^2 30^\circ} = \tan 30^\circ \rightarrow \alpha = 30^\circ$

Jadi muka air sejajar dg bid. luncur

S.5.4

Catatan :

Jika suatu tangki dipercepat tanpa ada air yg tumpah maka :



• PBEP : $a_x dx + a_y dy + a_z dz = 0$ ($a_y = 0$)

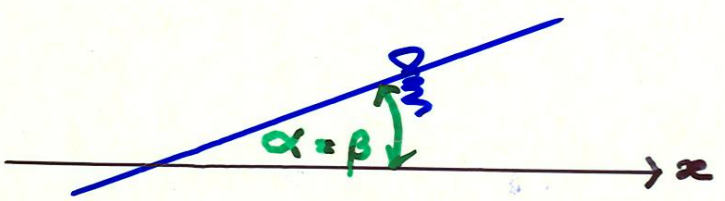
$\therefore$ arah muka air → $\tan\alpha = -\frac{a_x}{a_z}$ ..... (1)

• Menurut gambar $\tan\alpha = \tan(180-\beta)$

$= -\tan\beta = -\frac{a}{b}$ ..... (2)

• (1) = (2), shg $\boxed{\frac{a}{b} = \frac{a_x}{a_z}}$ ← a_x (−), a_z (−)

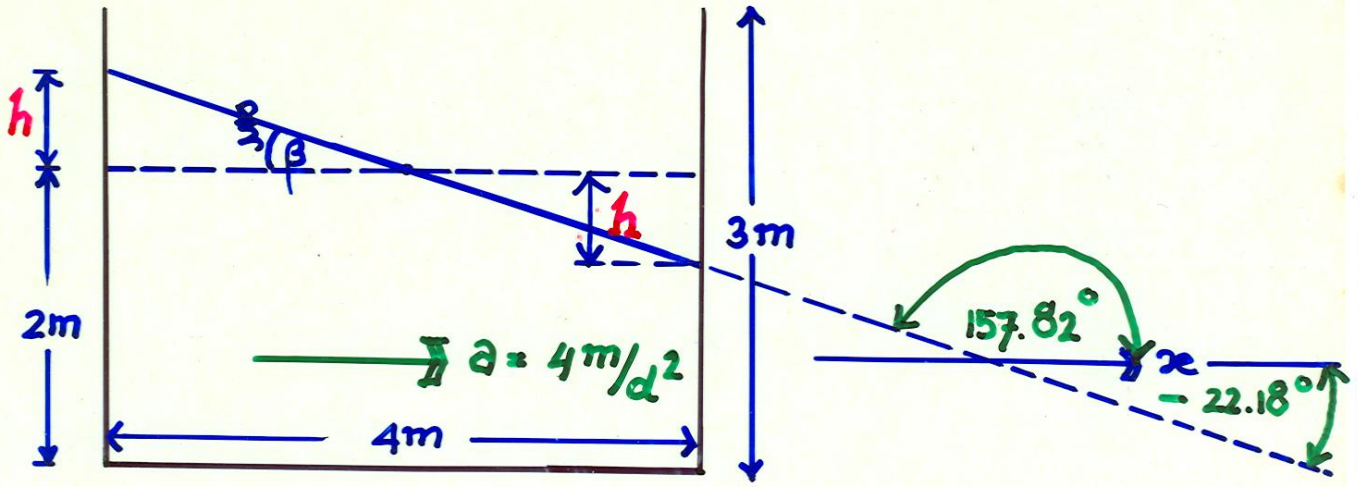
● Jika tangki dipercepat kekiri maka



• $\tan\alpha = \tan\beta = \frac{a}{b}$ ..... (3)

• (1) = (3), shg $\boxed{\frac{a}{b} = -\frac{a_x}{a_z}}$ ← a_x (+), a_z (−)

S.5.5



a) Arah m.a. : $\tan\alpha = -\frac{a_x}{a_z} = -\frac{-4\ m/d^2}{-9.81\ m/d^2} = -0.408$

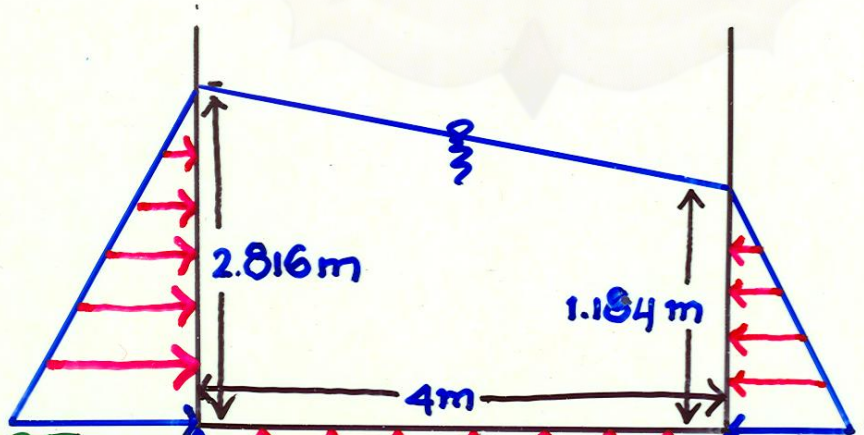
$\therefore \alpha = -22°10'59''$ atau

$\alpha = 180° - 22°10'59'' = 157°49'01''$

b) $h = (\frac{1}{2}\cdot 4)\tan\beta = (2)(0.408) = 0.816\ m$

Jadi tinggi m.a. kiri = 2+0.816 = 2.816 m

tinggi m.a. kanan = 2-0.816 = 1.184 m.



$p = 2.816\ T/m^2$

$p = 2.816\ T/m^2$

$p = 1.184\ T/m^2$

h = γ_{air}

$p = 1.184\ m \times 1\ T/m^3$

$= 1.184\ T/m^2$

S.5.6

Soal 4, hal. 101, SP Hidraulika I

Suatu tangki bergerak dg percepatan $3\ m/det^2$ dg arah miring kekanan atas membentuk sudut 30° dg bid. horisontal. Jika tangki berisi air, hitung kemiringan muka air

◇ Percepatan cairan

$a_c \cos\beta = a_c \frac{1}{2}\sqrt{3} = 1.5\sqrt{3}\ m/det^2$

$a_c \sin\beta = 0.5\, a_c = 1.5\ m/det^2$

$a_c = 3\ m/det^2$

$g = 9.81\ m/det^2$

◇ Percepatan tangki

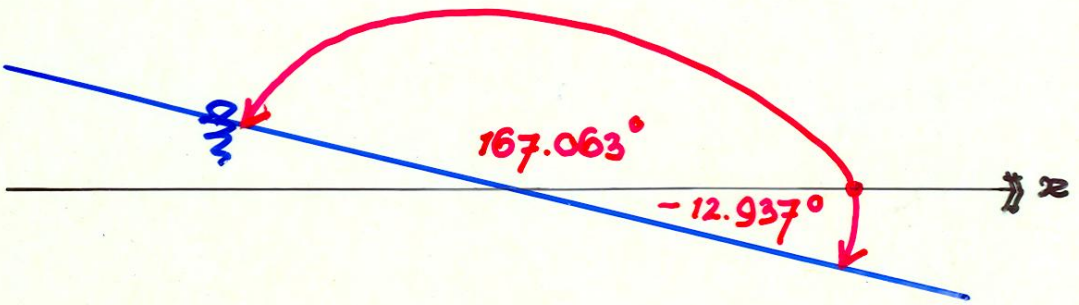
$a_t = 3\ m/det^2$

$\beta = 30°$

◇ Arah muka air :

$$\tan\alpha = -\frac{a_x}{a_z} = -\frac{-1.5\sqrt{3}}{-1.5 - 9.81} = -0.2297$$

Jadi : $\alpha = -12.937°$ atau $167.063°$

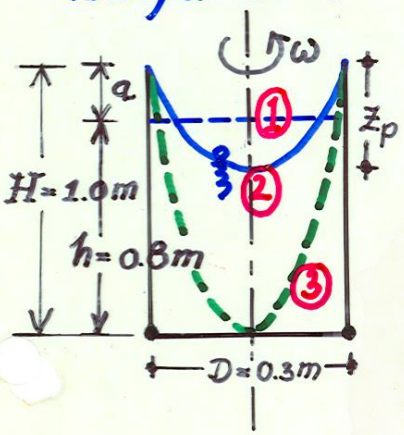


S.5.7

Tangki silinder terbuka dengan tinggi $H = 1.0$ m dan diameter $D = 0.3$ m berisi air dg kedalaman $h = 0.8$ m. Silinder diputar terhadap sumbunya.

- Berapa kecep. sudut max. agar air tidak tumpah ?
- Berapa kecep. sudutnya agar kedalaman air di sumbu silinder adalah nol ?
- Berapa air yang tumpah pada keadaan 2.

Penyelesaian :



- Luas alas silinder : $A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi . 0.3^2$
  $= 0.071\ m^2$
- Volume air semula: $\forall = A \times h = 0.071 \times h$
  $= 0.057\ m^3$
- Pada kedudukan ② air belum tumpah :

$$z_p = 2a = 2(H-h) = 2 \times 0.2 = 0.40\ m$$

$$z_p = \frac{\omega^2 R^2}{2g} \rightarrow \omega = \sqrt{\frac{2g z_p}{R^2}} = \sqrt{\frac{2(9.81)(0.4)}{0.15^2}}$$

sehingga $\omega = 18.676$ radian/detik

- Pada kedudukan ③ :

$$z_p = H = 1.0\ m \rightarrow \omega = \sqrt{\frac{2(9.81)(1.0)}{0.15^2}} = 29.53\ \text{rad/detik}$$

Volume air yg tinggal : $\forall_{akhir} = \frac{1}{2} \forall_{silinder\ total}$

$$= \frac{1}{2} \times A \times H$$

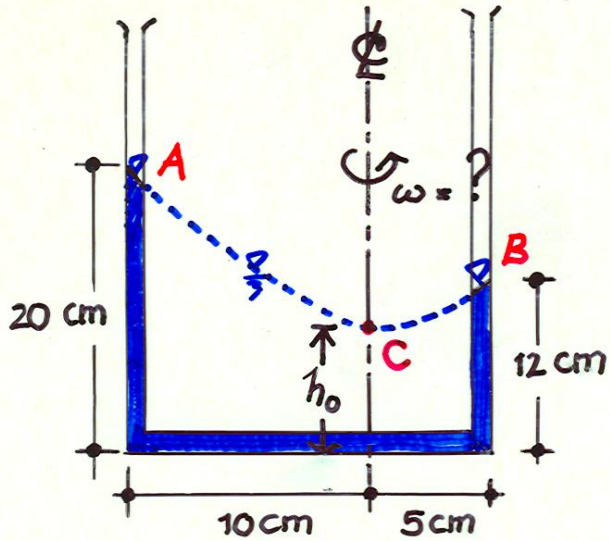
$$= \frac{1}{2} \times 0.071 \times 1.0 = 0.035\ m^3$$

- Jadi volume air yg tumpah :

$$\forall_{tumpah} = \forall_{semula} - \forall_{akhir}$$

$$= 0.057 - 0.035 = 0.22\ m^3$$

9.5.8



Persamaan muka air :

$$z = \frac{\omega^2}{2g} x^2 + h_o \qquad \text{melalui ttk A dan B}$$

sehingga

$$20 = \frac{\omega^2}{2g} 10^2 + h_o \qquad \text{(mell. A)}$$

$$12 = \frac{\omega^2}{2g} 5^2 + h_o \qquad \text{(mell. B)}$$

$$8 = 75 \frac{\omega^2}{2g}$$

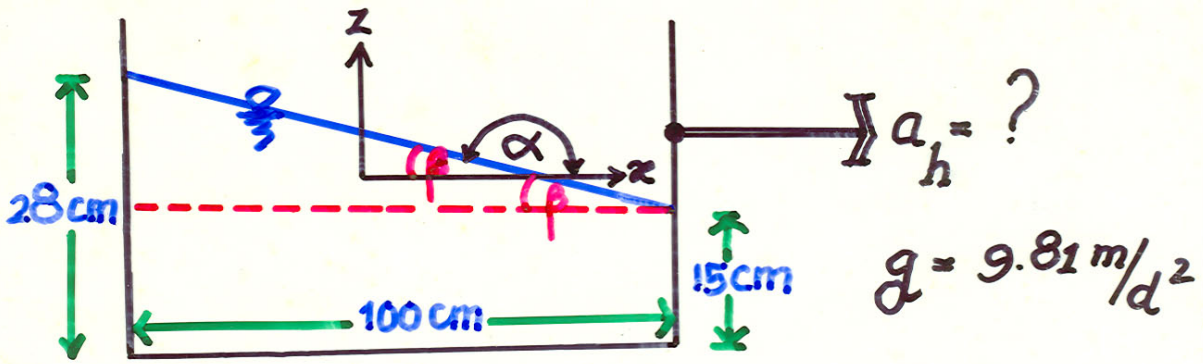
$$\omega = \sqrt{\frac{2 \times 981 \times 8}{75}}$$

$$\omega = 14.46 \text{ rad/det} \rightarrow N = \frac{60 \times 14.46}{2\pi} = 138 \text{ rpm}$$

$$h_o = 20 - \frac{14.46^2}{2 \times 981} 10^2$$

$$= 9.34 \text{ cm}$$

S.5.1



Jawaban :

- Pers. Bid. Equi. Potensial (PBEP) :

$$a_x dx + a_y^{=0} dy + a_z dz = 0$$

$$z = -\frac{a_x}{a_z} x + C \rightarrow \text{arah grs} : \tan\alpha = -\frac{a_x}{a_z}$$

- Diketahui : ⇒ $\tan\beta = \frac{28-15}{100} = 0.13$

⇒ $a_x = -a_h$

⇒ $a_z = -g = -9.81\ m/det^2$

- Dari gbr tampak :

$$\alpha + \beta = 180 \rightarrow \beta = 180 - \alpha$$

$$\therefore \tan\beta = \tan(180-\alpha)$$

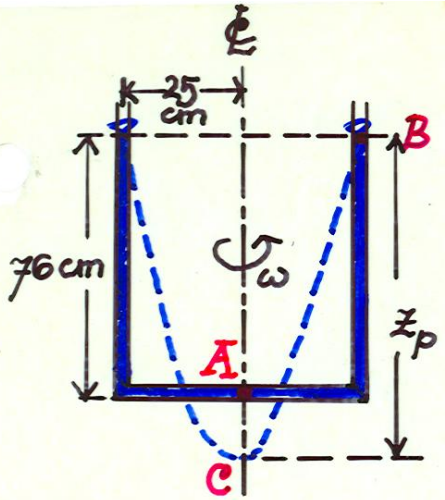
$$0.13 = -\tan\alpha$$

$$= \frac{a_x}{a_z}$$

$$= \frac{-a_h}{-9.81} \rightarrow$$

Jadi $a_h = 0.13 \times 9.81$

$= 1.274\ m/det^2$ (kekanan)



S.5.9

Sebuah silinder U dgn radius 25 cm, berisi air raksa setinggi 76 cm, diputar pada sumbunya 180 rpm. Diameter kaki silinder sangat kecil. Hitung tekanan di titik A dan B sebelum dan sesudah diputar, jika diketahui tekanan udara luar 10331 kg_f/m^2 dan $\gamma_{Hg} = 13568\ kg_f/m^3$ serta $g = 9.81\ m/det^2$.

- Sebelum & pd waktu diputar tekanan di ttk B sama dengan tekanan udara luar yaitu 10.331 Ton/m^2.

- Sebelum diputar tekanan di A adalah

$$p_A = h_A \times \gamma = 0.76 \times 13.568\ T/m^2 + atm$$
$$= 10.312\ T/m^2 + 10.331\ T/m^2$$
$$\therefore \boxed{p_A = 20.643\ T/m^2}$$

- Tekanan di A setelah diputar dg $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 180}{60} = 18.85\ r/s$

$$z_p = \frac{\omega^2 R^2}{2g} = \frac{18.85^2 \times 0.25^2}{2 \times 9.81} = 1.132\ m$$

- Menurut PBEP maka

$$p_B = p_C = p_A + h_{AC}\gamma_{Hg}$$

Jadi $p_A = p_B - h_{AC}\gamma_{Hg}$

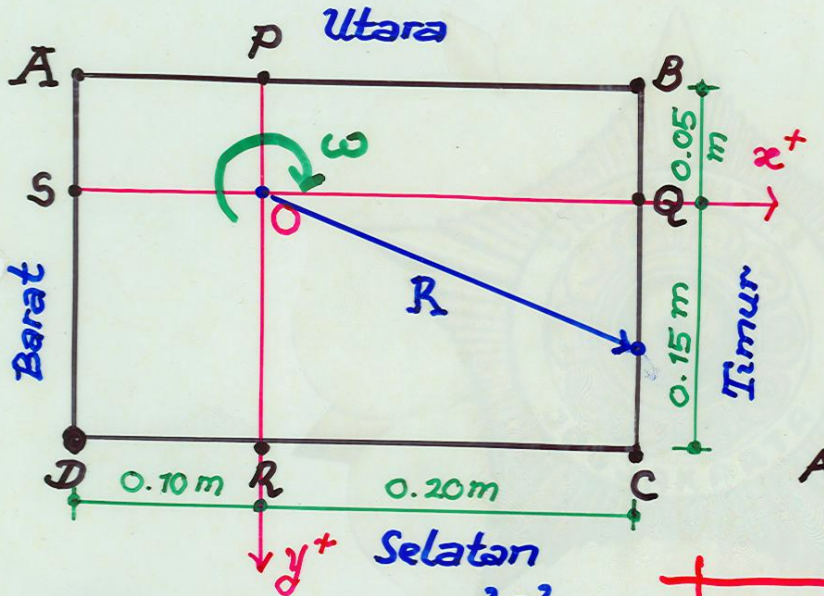
$$= 10.331 - (1.132 - 0.76)\ 13.568$$

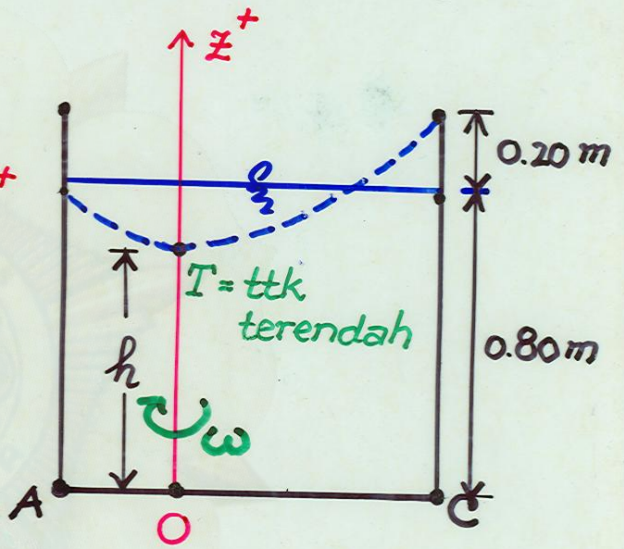
$$\boxed{p_A = 5.284\ Ton/m^2}$$

5.5.10

Sebuah tangki dgn dasar segiempat mempunyai dinding vertikal setinggi 1.0 m dan berisi air sedalam 0.8 m. Tangki ini diputar dengan sumbu putar pada grs vertikal melalui O. ($g = 9.81\ m/det^2$)

1. Berapa kecepatan sudut putar maksimum agar air belum tumpah?
2. Tentukan persamaan & gambar muka air sepanjang dinding!
3. Berapa gaya hidrostatika yg bekerja pada dinding?





1. PBEP : $z_p = h + \frac{\omega^2 R^2}{2g}$ atau $z = h + \frac{\omega^2}{2g}(x^2+y^2)$ .....(1)

Vol. paraboloida yg dibatasi dinding tangki dpt dihitung dgn : $\forall = \int_{y_1}^{y_2}\int_{x_1}^{x_2} z\,dx\,dy$

$$\forall(x_1, x_2, y_1, y_2, \omega, g, h) = h(x_1-x_2)(y_1-y_2) + \frac{\omega^2}{6g}(x_1-x_2)(y_1-y_2)(x_1^2+x_1x_2+x_2^2+y_1^2+y_1y_2+y_2^2)$$

.....(2)

S.5.11

o) Muka air tertinggi ada di atas C → air belum tumpah

Pers.(1): $z_c = h + \frac{\omega^2}{2g}(0.15^2 + 0.20^2)$ .....(3)

$z_c = 1.0\ m$ .....(4)

Pers.(2): Vol. air $= 0.2 \cdot 0.3 \cdot 0.8 = 0.048\ m^3$ ....(5)

Vol. air $= \forall(-0.1, 0.2, -0.05, 0.15, \omega, g)$

$= 0.06h + 0.000475\frac{\omega^2}{g}$ ....(6)

Dari Pers.(3) s/d (6) dan nilai $g = 9.81\ m/det^2$, maka diperoleh:

$h = 0.732\ m$

$\omega = 9.17\ rad/det = 87.57\ rpm$

2) Pers. muka air di dinding: pers. parabola

o) Utara, $y = -0.05\ m$

$z = h + \frac{\omega^2}{2g}(0.05^2 + x^2) = 0.732 + 4.286(0.05^2 + x^2)$

Jadi $z = 0.743 + 4.286x^2$ → $z_A = 0.743 + 4.286 \times 0.1^2 = 0.786\ m$

$z_B = 0.743 + 4.286 \times 0.2^2 = 0.914\ m$

o) Timur, $x = 0.20\ m$

Jadi $z = 0.904 + 4.286y^2$ → $z_B = 0.904 + 4.286 \times (-0.05)^2 = 0.914\ m$

$z_C = 0.904 + 4.286 \times 0.15^2 = 1.00\ m$

o) Selatan, $y = 0.15\ m$

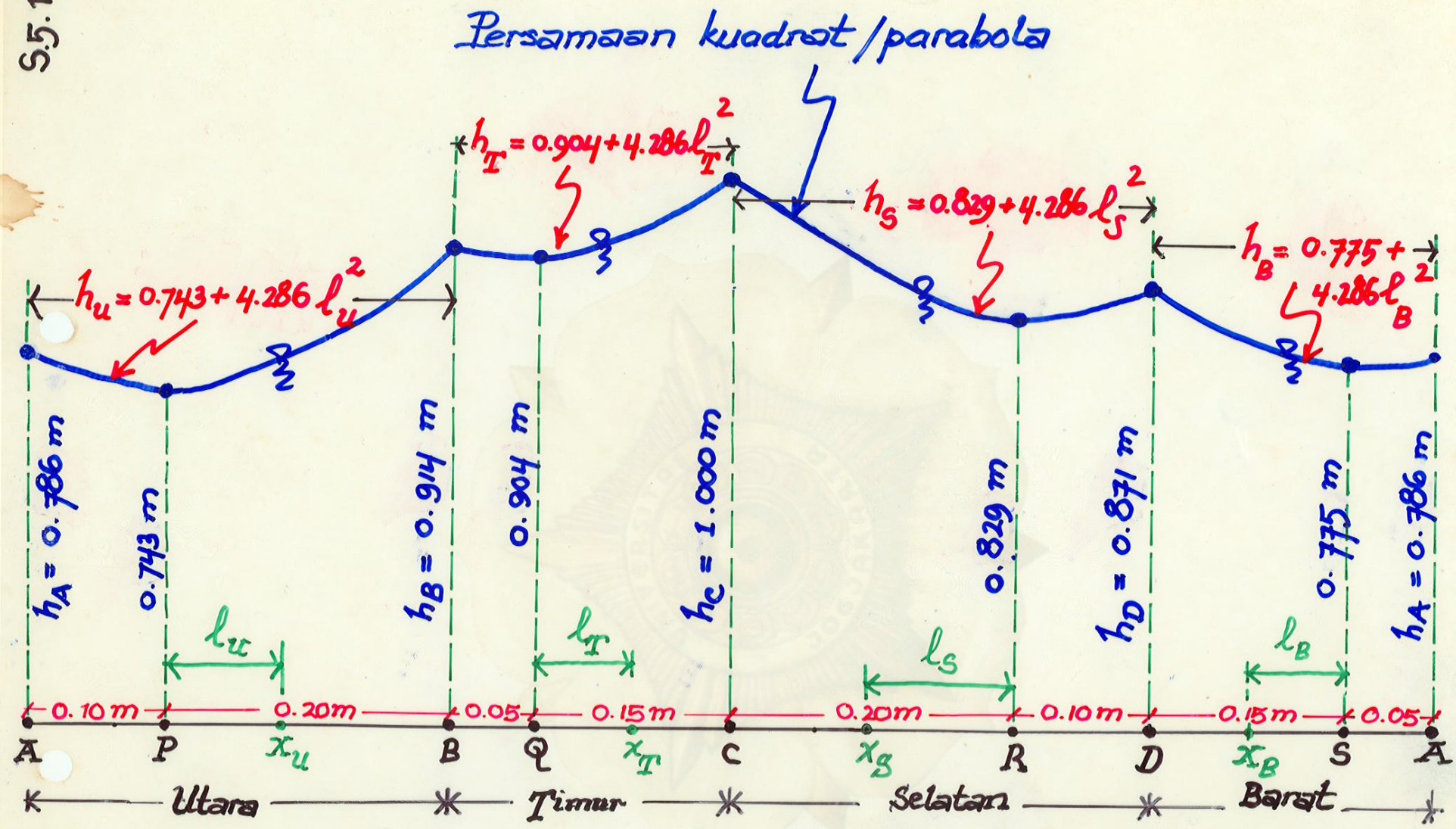
Jadi $z = 0.829 + 4.286x^2$ → $z_C = 0.829 + 4.286 \times 0.20^2 = 1.00\ m$

$z_D = 0.829 + 4.286 \times 0.10^2 = 0.871\ m$

o) Barat, $x = -0.10\ m$

Jadi $z = 0.775 + 4.286y^2$ → $z_D = 0.775 + 4.286 \times 0.15^2 = 0.871\ m$

$z_A = 0.775 + 4.286(-0.05)^2 = 0.786\ m$



3. Gaya hidrostatika yg bekerja di dasar tangki pd saat diputar
   ≈ gaya hidrostatika pd saat air diam
   ≈ berat air = 0.20 m × 0.3 m × 0.8 m × 1 T/m^3 = **0.048 Ton_f**