Metoda Numerik

Aplikasi di Lapangan: Kuda-kuda Asimetri

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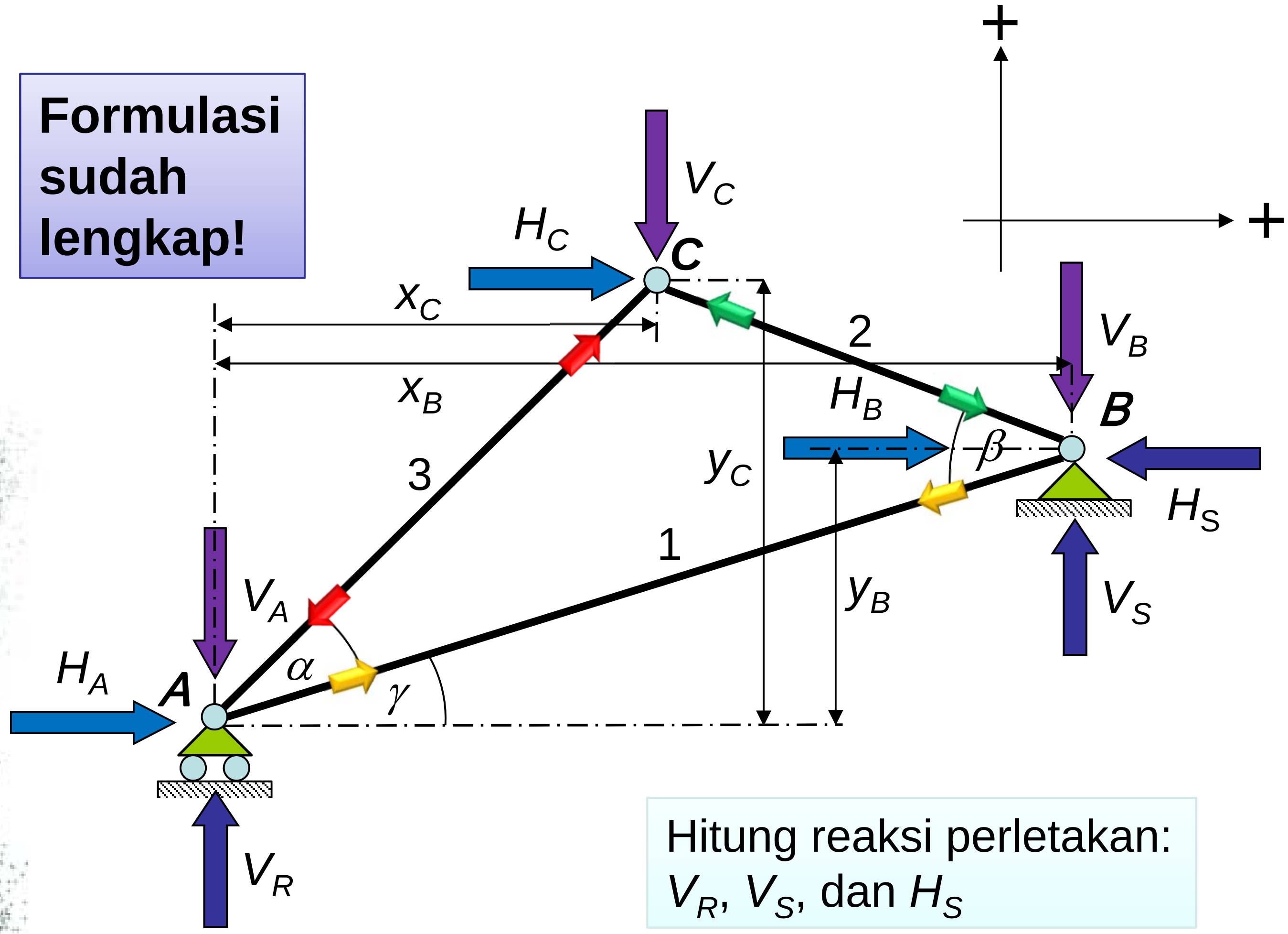
Kuda-kuda Asimetri

Formulasi sudah lengkap!



Hitung reaksi perletakan:
V_R, V_S, dan H_S

Hitung gaya-gaya internal yaitu gaya-gaya batang:
P_1, P_2, dan P_3

Hitungan Reaksi Perletakan dan Gaya Batang

- Diketahui:
  x_B, x_C, y_B, y_C , $V_A, H_A, V_B, H_B, V_C, H_C$

- Dihitung:
  $V_R, V_S, H_S, P_1, P_2, P_3$

- Reaksi perletakan dihitung dengan cara:
  - $\Sigma F_H = 0$
  - $\Sigma F_V = 0$
  - $\Sigma M_B = 0$

Hitungan Reaksi Perletakan

Reaksi perletakan dihitung dengan cara:

$$1. \Sigma F_H = 0 \rightarrow H_A + H_B + H_C + H_s = 0$$

$$H_s = -H_A - H_B - H_C$$

$$2. \Sigma F_V = 0 \rightarrow V_R + V_S + V_A + V_B + V_C = 0$$

$$V_R + V_S = -V_A - V_B - V_C$$

$$3. \Sigma M_B = 0 \rightarrow (V_R + V_A)\, x_B - H_A y_B + H_C\, (y_C - y_B) + V_C\, (x_B - x_C) = 0$$

$$V_R\, x_B = H_A y_B - H_C\, (y_C - y_B) - V_C\, (x_B - x_C) - V_A y_B$$

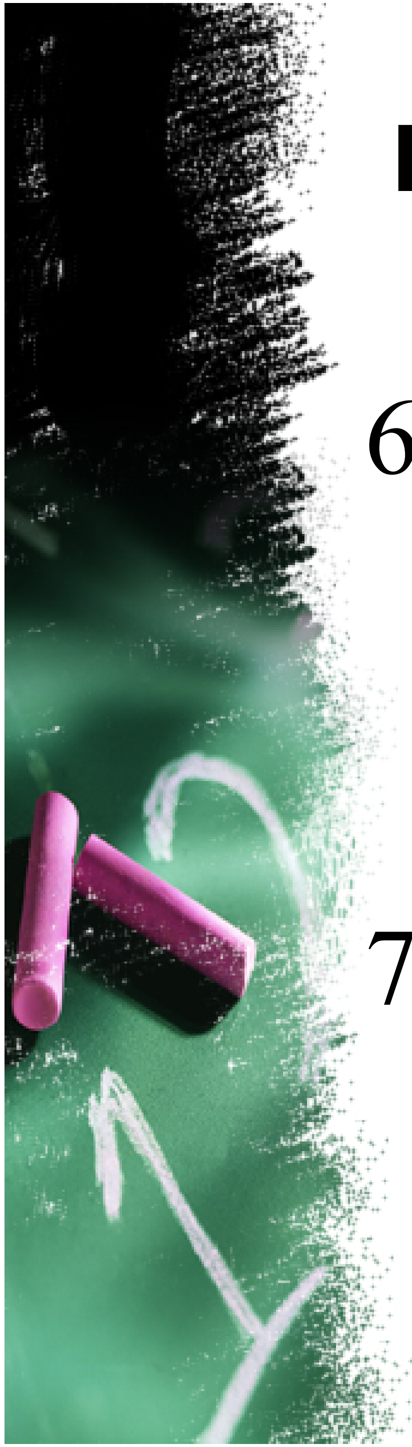
Kesetimbangan Titik buhul *A*

$$4.\ \Sigma F_H = 0$$

- $P_1 \cos \gamma - P_3 \cos(\alpha + \gamma) + H_A = 0$
- $-P_1 \cos \gamma + P_3 \cos(\alpha + \gamma) = H_A$

$$5.\ \Sigma F_V = 0$$

- $V_R + V_A + P_1 \sin \gamma - P_3 \sin(\alpha + \gamma) = 0$
- $-P_1 \sin \gamma + P_3 \sin(\alpha + \gamma) = V_A + V_R$



Kesetimbangan Titik buhul *B*

$$6.\ \Sigma F_H = 0$$

- $-P_1 \cos\gamma + P_2 \cos(\beta - \gamma) + H_B + H_S = 0$
- $P_1 \cos\gamma - P_2 \cos(\beta - \gamma) = H_B + H_S$

$$7.\ \Sigma F_V = 0$$

- $V_B + V_S - P_1 \sin\gamma - P_2 \sin(\beta - \gamma) = 0$
- $P_1 \sin\gamma + P_2 \sin(\beta - \gamma) = V_B + V_S$

Kesetimbangan Titik buhul *C*

8. $\Sigma F_H = 0$

- $-P_2 \cos(\beta - \gamma) + P_3 \cos(\alpha + \gamma) + H_C = 0$
- $P_2 \cos(\beta - \gamma) - P_3 \cos(\alpha + \gamma) = H_C$

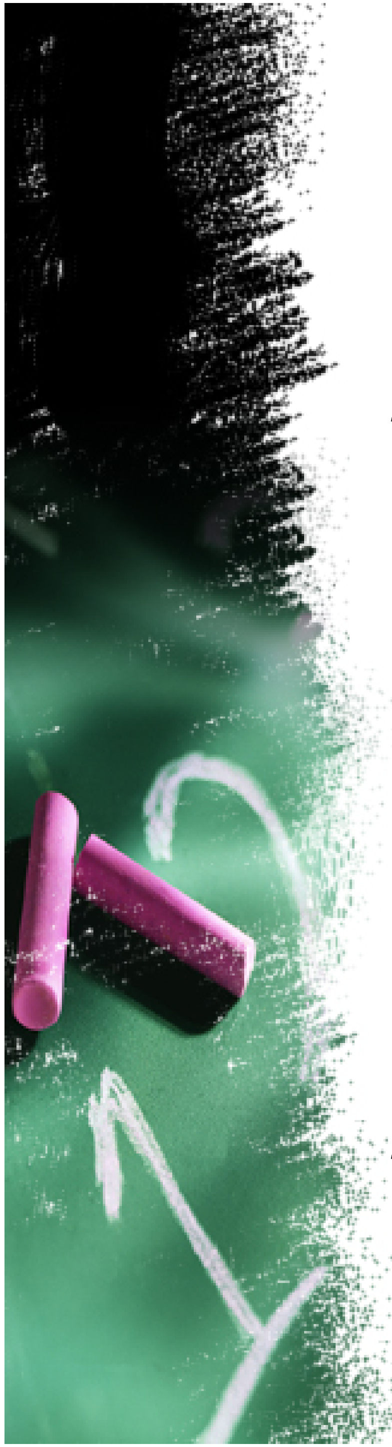
9. $\Sigma F_V = 0$

- $P_2 \sin(\beta - \gamma) + P_3 \sin(\alpha + \gamma) + V_C = 0$
- $-P_2 \sin(\beta - \gamma) - P_3 \sin(\alpha + \gamma) = V_C$

Sistem persamaan linier

Dipilih 6 dari 9 persamaan yang dibutuhkan untuk menghitung: V_R, V_S, H_S, P_1, P_2, P_3

1. $V_R = \{H_A y_B - H_C\,(y_C - y_B) - V_C\,(x_B - x_C) - V_A\,x_B\}/x_B$
2. $V_R + V_S = -V_A - V_B - V_C$
3. $H_s = -H_A - H_B - H_C$
4. $P_1 \cos\gamma - P_2 \cos(\beta - \gamma) = H_B + H_S$
5. $P_2 \cos(\beta - \gamma) - P_3 \cos(\alpha + \gamma) = H_C$
6. $-P_2 \sin(\beta - \gamma) - P_3 \sin(\alpha + \gamma) = V_C$



Modifikasi sistem persamaan linier

1. $V_R = \{H_A y_B – H_C\,(y_C\text{-}y_B) – V_C\,(x_B\text{-}x_C) – V_A\,x_B\}/x_B$
2. $V_R + V_S = – V_A – V_B – V_C$
3. $H_s = – H_A – H_B – H_C$
4. $– P_1 \cos \gamma – P_2 \cos (\beta - \gamma) = H_B + H_S$
5. $– P_2 \cos (\beta – \gamma) + P_3 \cos (\alpha + \gamma) = H_C$
6. $– P_2 \sin (\beta – \gamma) – P_3 \sin (\alpha + \gamma) = V_C$

Reaksi perletakan dihitung terlebih dahulu:

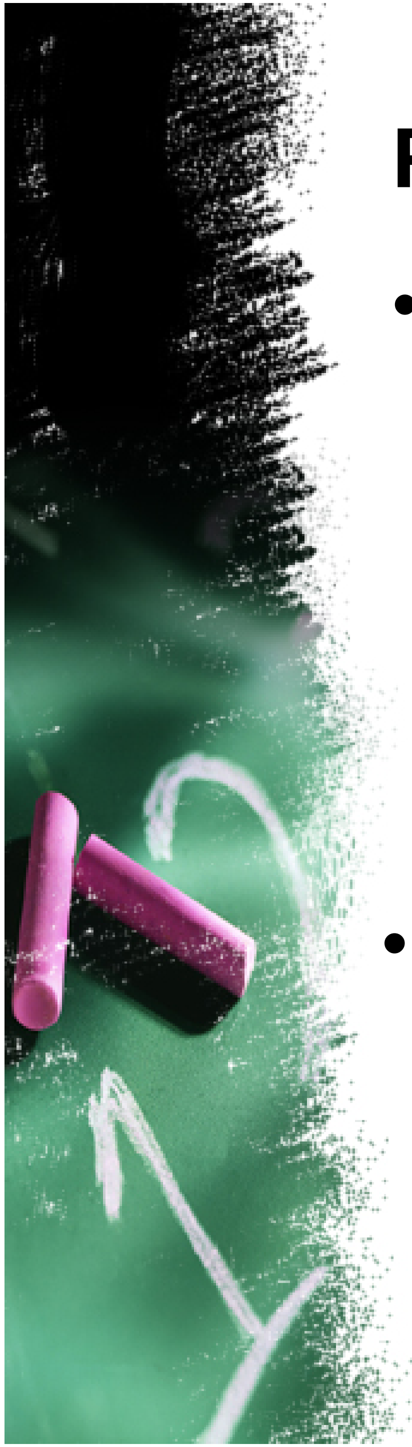
1. $V_R = \{H_A y_B – H_C\,(y_C - y_B) + V_C x_C\}/x_B – V_A – V_C$
2. $V_S = – V_A – V_B – V_C – V_R$
   $V_S = – \{H_A y_B – H_C\,(y_C - y_B) + V_C x_C\}/x_B – V_B$
3. $H_s = – H_A – H_B – H_C$

Sistem persamaan linier final

1. $V_R = \{H_A y_B - H_C\,(y_C - y_B) + V_C x_C\}/x_B - V_A - V_C$
2. $V_S = -\{H_A y_B - H_C\,(y_C - y_B) + V_C x_C\}/x_B - V_B$
3. $H_s = -H_A - H_B - H_C$
4. $P_1 \cos\gamma - P_2 \cos(\beta - \gamma) = H_B + H_S$
5. $P_2 \cos(\beta - \gamma) - P_3 \cos(\alpha + \gamma) = H_C$
6. $-P_2 \sin(\beta - \gamma) - P_3 \sin(\alpha + \gamma) = V_C$

Persamaan 4, 5, dan 6 diselesaikan untuk menghitung: P_1, P_2, P_3

$$\begin{bmatrix} \cos\gamma & -\cos(\beta-\gamma) & 0 \\ 0 & \cos(\beta-\gamma) & -\cos(\alpha+\gamma) \\ 0 & -\sin(\beta-\gamma) & -\sin(\alpha+\gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$



Rekapitulasi Hitungan

- Reaksi perletakan

$$V_R = \frac{1}{x_B}\left\{H_A y_B - H_C(y_C - y_B) + V_C x_C\right\} - V_A - V_C$$

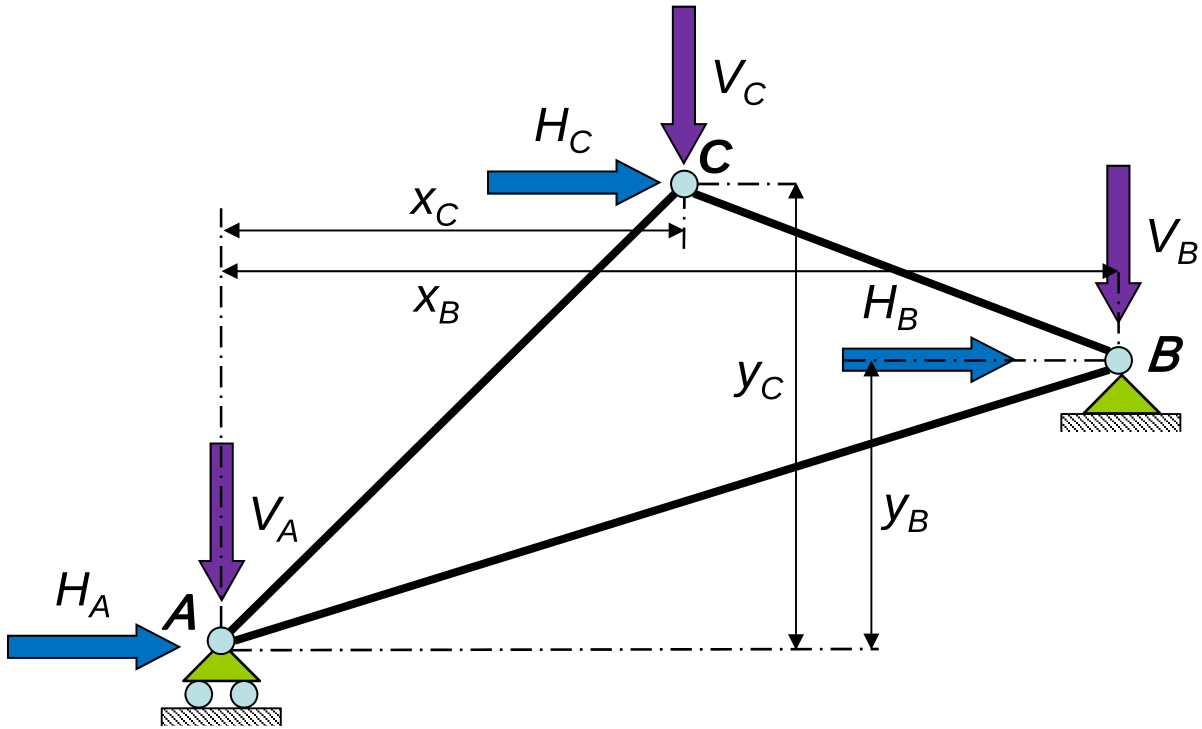
$$V_S = \frac{-1}{x_B}\left\{H_A y_B - H_C(y_C - y_B) + V_C x_C\right\} - V_B$$

$$H_S = -H_A - H_B - H_C$$

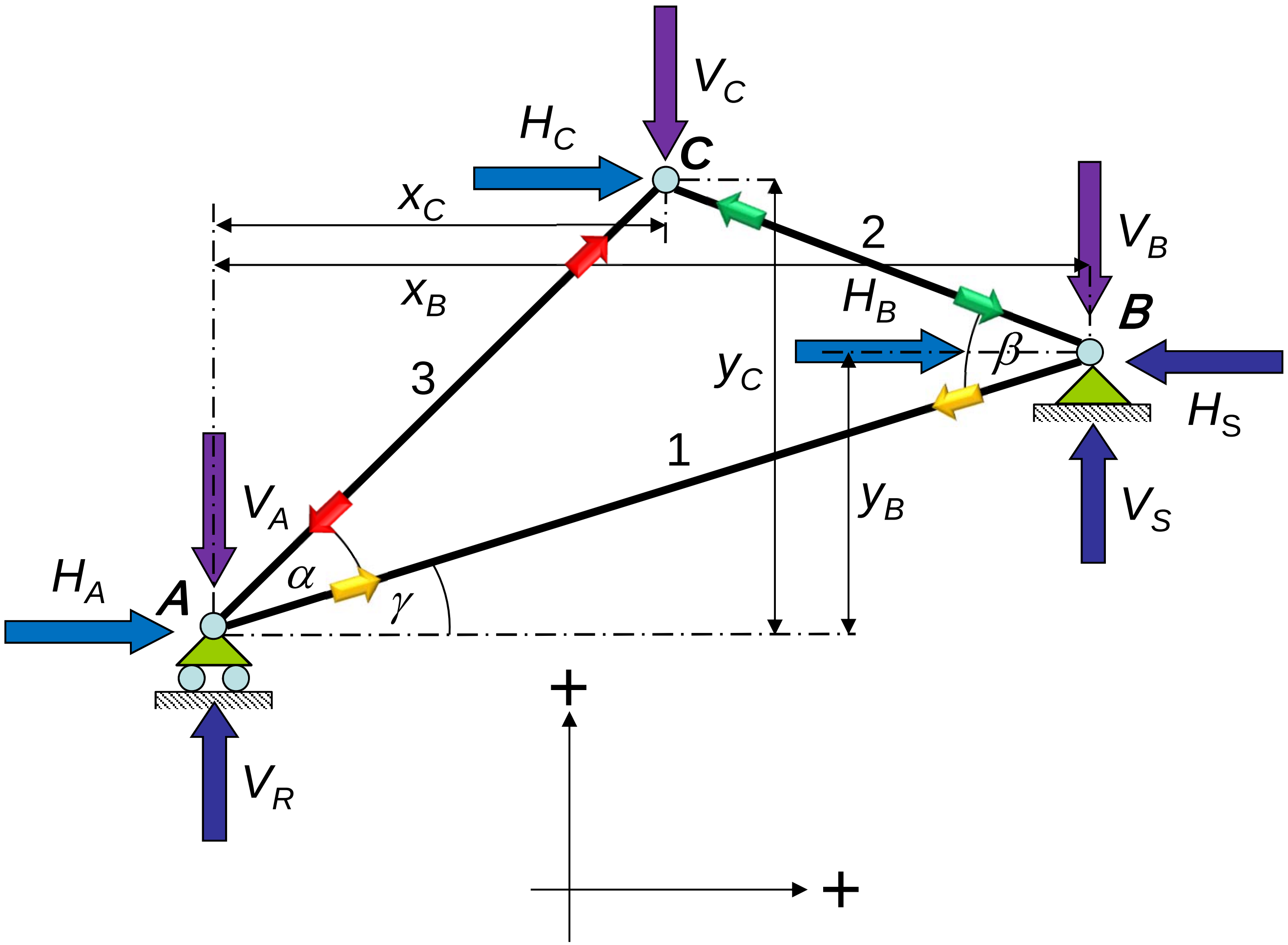
- Gaya batang

$$\begin{bmatrix} \cos\gamma & -\cos(\beta-\gamma) & 0 \\ 0 & \cos(\beta-\gamma) & -\cos(\alpha+\gamma) \\ 0 & -\sin(\beta-\gamma) & -\sin(\alpha+\gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$

Gambar Soal



Gambar Jawaban



Eliminasi Gauss 1/2

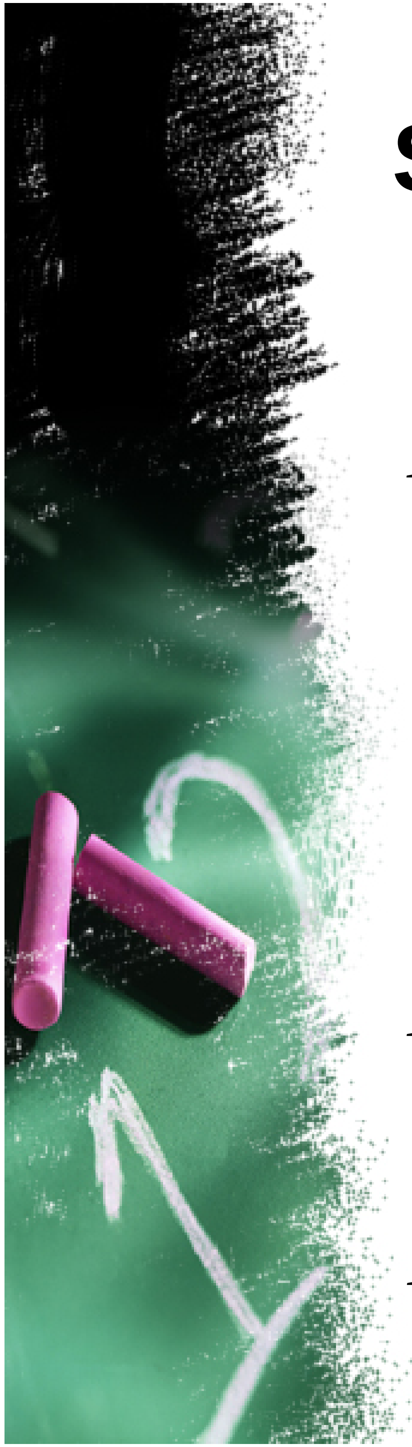
$$\begin{bmatrix} \cos\gamma & -\cos(\beta-\gamma) & 0 \\ 0 & \cos(\beta-\gamma) & -\cos(\alpha+\gamma) \\ 0 & -\sin(\beta-\gamma) & -\sin(\alpha+\gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$

$$\begin{bmatrix} 1 & -\dfrac{\cos(\beta-\gamma)}{\cos\gamma} & 0 \\ 0 & 1 & \dfrac{-\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} \\ 0 & 1 & \dfrac{\sin(\alpha+\gamma)}{\sin(\beta-\gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \dfrac{H_B + H_S}{\cos\gamma} \\ \dfrac{H_C}{\cos(\beta-\gamma)} \\ \dfrac{V_C}{-\sin(\beta-\gamma)} \end{Bmatrix}$$

Eliminasi Gauss 2/2

$$\begin{bmatrix} 1 & -\dfrac{\cos(\beta-\gamma)}{\cos\gamma} & 0 \\ 0 & 1 & \dfrac{-\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} \\ 0 & 1 & \dfrac{\sin(\alpha+\gamma)}{\sin(\beta-\gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \dfrac{H_B + H_S}{\cos\gamma} \\ \dfrac{H_C}{\cos(\beta-\gamma)} \\ \dfrac{V_C}{-\sin(\beta-\gamma)} \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -\dfrac{\cos(\beta-\gamma)}{\cos\gamma}\dfrac{\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} \\ 0 & 1 & \dfrac{-\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} \\ 0 & 0 & \dfrac{\sin(\alpha+\gamma)}{\sin(\beta-\gamma)} + \dfrac{\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \dfrac{H_B + H_C + H_S}{\cos\gamma} \\ \dfrac{H_C}{\cos(\beta-\gamma)} \\ \dfrac{V_C}{-\sin(\beta-\gamma)} - \dfrac{H_C}{\cos(\beta-\gamma)} \end{Bmatrix}$$



Substitusi

$$P_3 = \frac{\dfrac{V_C}{-\sin(\beta-\gamma)} - \dfrac{H_C}{\cos(\beta-\gamma)}}{\dfrac{\sin(\alpha+\gamma)}{\sin(\beta-\gamma)} + \dfrac{\cos(\alpha+\gamma)}{\cos(\beta-\gamma)}}$$

$$= -\frac{V_C + H_C \tan(\beta-\gamma)}{\sin(\alpha+\gamma) + \tan(\beta-\gamma)\cos(\alpha+\gamma)}$$

$$P_2 = \frac{H_C}{\cos(\beta-\gamma)} + \frac{\cos(\alpha+\gamma)}{\cos(\beta-\gamma)} P_3$$

$$P_1 = \frac{H_B + H_C + H_S}{\cos\gamma} + \frac{\cos(\alpha+\gamma)}{\cos\gamma} P_3$$