Jawaban Soal

I.1.a. $x_A = 451.023 \qquad x_T = 451.01$

angka signifikan pertama (a.s.p.)

$|x_A - x_T| = 0.013$

angka signifikan terakhir (a.s.t.)

a.s. = 4

b. $x_A = -0.045113 \qquad x_T = -0.04518$

asp

$|x_A - x_T| = 0.000067$

ast

a.s. = 2

c. $x_A = 23.4213 \qquad x_T = 23.4604$

asp

$|x_A - x_T| = 0.0391$

ast

a.s. = 3

2. Pers. kuadrat : $x^2 - 26x + 1 = 0$

$$r_T = \frac{26 \pm \sqrt{26^2 - 4}}{2} = 13 \pm \sqrt{168} \doteq 13 \pm 12.9614814$$

$$r_T^{(1)} \doteq 25.9614814 \qquad r_T^{(2)} \doteq 0.0385186$$

asp asp

Dengan menggunakan 5 a.s. : $\sqrt{168} = 12.961$, maka

$$r_A^{(1)} = 25.961 \qquad r_A^{(2)} = 0.039$$

$|r_T^{(1)} - r_A^{(1)}| = 0.0004814 \rightarrow$ a.s. = 5 $\rightarrow$ OK

ast

$|r_T^{(2)} - r_A^{(2)}| = 0.0004814 \rightarrow$ a.s. = 2 $\rightarrow$ not OK

$r_A^{(2)}$ harus dihitung dengan :

$$r_A^{(2)} = \frac{(13 - \sqrt{168})(13 + \sqrt{168})}{13 + \sqrt{168}} = \frac{13^2 - 168}{13 + 12.961} = 0.0385193$$

Check : $|r_T^{(2)} - r_A^{(2)}| = 0.00000072 \rightarrow$ a.s. = 4 dianggap OK
ast

II.1. Persamaan : $f(x) \equiv x^6 - x - 1$, $\alpha = 1.13472413840152$

n	x_n	$\alpha - x_n$	$r = \frac{\lvert\alpha - x_{n+1}\rvert}{\lvert\alpha - x_n\rvert}$	$f(x_n)$
	a = 1.0			61.0
	b = 2.0			-1.0
1	1.5	-0.36528	-	8.89063
2	1.25	-0.11528	0.31559	1.56470
3	1.125	0.00972	0.08432	-0.09771
4	1.1875	-0.05278	5.43004	0.61665
5	1.15625	-0.02153	0.40792	0.23327
6	1.14063	-0.00591	0.27450	0.06158
7	1.13281	0.00191	0.32318	-0.01958
8	1.13672	-0.00200	1.04712	0.02062
9	1.13477	-0.00005	0.02500	0.00043
10	1.13379	0.00093	18.60000	-0.00960
11	1.13428	0.00044	0.47312	-0.00459
12	1.13452	0.00020	0.45455	-0.00208
13	1.13464	0.00008	0.40000	-0.00083
14	1.13470	0.00002	0.25000	-0.00020
15	1.13474	-0.00002	1.00000	0.00016

maka untuk $n = 15$

$$x_n = 1.13474$$

$$|\alpha - x_n| = 0.00002 < 0.00005$$

Jika diperhatikan : $|\alpha - x_n| \leq \left(\frac{1}{2}\right)^n (b-a)$

jadi metoda ini mempunyai derajad konvergensi linier = $\frac{1}{2}$

II.2. Persamaan $x^6 - x - 1 = 0$ diselesaikan dengan metoda Newton.

n	x_n	$f(x_n)$	$\alpha - x_n$	$x_{n+1} - x_n$
0	2.0	61.0	−8.653E−1	–
1	1.680628	19.85	−5.459E−1	−2.499E−1
2	1.430739	6.147	−2.960E−1	−1.758E−1
3	1.254971	1.652	−1.202E−1	−9.343E−2
4	1.161538	2.943E−1	−2.681E−2	−2.519E−2
5	1.136353	1.683E−2	−1.629E−3	−1.623E−3
6	1.134730	6.574E−5	−6.390E−6	−6.390E−6
7	1.134724	1.015E−9	−9.870E−11	−9.870E−11

Pada $n = 7 \rightarrow$ akarnya $x_n = 1.134724$
dimana $\alpha - x_n = -9.870E-11$

Dari Tabel di atas tampak bahwa $x_{n+1} - x_n$ merupakan nilai prakiraan untuk $\alpha - x_n$.

3. Pers : $f(x) \equiv x^6 - x - 1 = 0$ diselesaikan dengan metoda sekan

n	x_n	$f(x_n)$	$\alpha - x_n$	$x_n - x_{n-1}$
0	2.0	61.0	−8.65E−1	
1	1.0	−1.0	1.35E−1	−1.0
2	1.016129	−9.154E−1	1.19E−1	1.61E−2
3	1.190578	6.575E−1	−5.59E−2	1.74E−1
4	1.117656	−1.685E−1	−1.71E−2	−7.29E−2
5	1.132532	−2.244E−2	2.19E−3	1.49E−2
6	1.134817	9.536E−4	−9.27E−5	2.29E−3
7	1.134724	−5.066E−6	4.92E−7	−9.32E−5
8	1.134724	−1.135E−9	1.10E−10	4.92E−7

II.4. Polinomial Laguerre :

$$p(x) = 720 - 4320x + 5400x^2 - 2400x^3 + 450x^4 - 36x^5 + x^6$$

"True"	Cara 1	Cara 2	Cara 3
15.98287	15.98287	15.98279	15.98279
9.837467	9.837471	9.837469	9.837467
5.775144	5.775764	5.775207	5.775144
2.992736	2.991080	2.992710	2.992736
1.188932	1.190937	1.188932	1.188932
0.2228466	0.2219429	0.2228466	0.222846

Dari tabel di atas tampak bahwa Cara 2 lebih baik dari Cara 1, tetapi Cara 3 adalah cara yang terbaik.

Catatan :

Dari hal. II.6 : $p(x) = b_0 + (x - z)\,q(x)$

Jika z adalah salah satu akar dari $p(x) = 0$, maka $b_0 = 0$ maka untuk mencari akar selanjutnya cukup dengan mencari akar dari $q(x) = 0$.

Demikian pencarian akar dilakukan dgn mencari akar $q(x) = 0$ sampai semua akar $p(x) = 0$ didapat

$$p(x) = (x - z_1)\,q_1(x) \;\rightarrow\; z_1 \text{ akar } p(x) = 0$$

$$q_1(x) = (x - z_2)\,q_2(x) \;\rightarrow\; z_2 \text{ akar } q_1(x) = 0$$

$$q_2(x) = (x - z_3)\,q_3(x) \;\rightarrow\; z_3 \text{ akar } q_2(x) = 0$$

$\vdots$

dst

z_i & $q_i(x)$ dapat dicari dgn algoritma Polynew yang menghasilkan <u>Cara 1 dan Cara 2</u> !

II.4. Polinomial Laguerre :

$$p(x) = 720 - 4320x + 5400x^2 - 2400x^3 + 450x^4 - 36x^5 + x^6$$

"True"	Cara 1	Cara 2	Cara 3
15.98287	15.98287	15.98279	15.98279
9.837467	9.837471	9.837469	9.837467
5.775144	5.775764	5.775207	5.775144
2.992736	2.991080	2.992710	2.992736
1.188932	1.190937	1.188932	1.188932
0.2228466	0.2219429	0.2228466	0.222846

Dari tabel di atas tampak bahwa Cara 2 lebih baik dari Cara 1, tetapi Cara 3 adalah cara yang terbaik.

Catatan :

Dari hal. II.6 : $p(x) = b_0 + (x-z)\,q(x)$

Jika z adalah salah satu akar dari $p(x)=0$, maka $b_0=0$ maka untuk mencari akar selanjutnya cukup dengan mencari akar dari $q(x)=0$.

Demikian pencarian akar dilakukan dgn mencari akar $q(x)=0$ sampai semua akar $p(x)=0$ didapat

$$p(x) = (x-z_1)\,q_1(x) \quad \rightarrow z_1 \text{ akar } p(x)=0$$

$$q_1(x) = (x-z_2)\,q_2(x) \quad \rightarrow z_2 \text{ akar } q_1(x)=0$$

$$q_2(x) = (x-z_3)\,q_3(x) \quad \rightarrow z_3 \text{ akar } q_2(x)=0$$

$\vdots$

dst

z_i & $q_i(x)$ dapat dicari dgn algoritma Polynew yang menghasilkan Cara 1 dan Cara 2 !

Sedangkan Cara 3 adalah dengan menggunakan hasil dari Cara 1 sebagai nilai awal untuk mencari akar polinomial asli $p(x)$ bukan $q_{h_i}(x)$!

III.1.a.

i	x_i	f	$f_1[\,]$	$f_2[\,]$	$f_3[\,]$
0	0.	(-5.)			
			(6.)		
1	1.	1.		(2.)	
			12.		(1.)
2	3.	25.		6.	
			30.		
3	4.	55.			

$$
\begin{aligned}
p_3(x) &= f(x_0) + (x-x_0)f[x_0,x_1] \\
&\quad + (x-x_0)(x-x_1)f[x_0,x_1,x_2] \\
&\quad + (x-x_0)(x-x_1)(x-x_3)f[x_0,x_1,x_2,x_3] \\
&= -5 + (x-0)(6) + (x-0)(x-1)(2) + (x-0)(x-1)(x-3)(1) \\
&= x^3 - 2x^2 + 7x - 5
\end{aligned}
$$

b.

i	x_i	f	$f_1[\,]$	$f_2[\,]$	$f_3[\,]$
3	0.	-5.			
			6.		
1	1.	1.		2.	
			(12.)		(1.)
0	3.	(25.)		(6.)	
			30.		
2	4.	55.			

$$
\begin{aligned}
p_3(x) &= 25 + (x-3)(12) + (x-3)(x-1)(6) + (x-3)(x-1)(x-4)(1) \\
&= x^3 - 2x^2 + 7x - 5
\end{aligned}
$$

c.

i	x_i	$f(x_i)$	$f_1[\,]$	$f_2[\,]$	$f_3[\,]$
0	3.	(25.)			
			(10.)		
1	0.	−5.		(5.)	
			15.		(1.)
2	4.	55.		3.	
			18.		
3	1.	1.			

$$P_3(x) = 25 + (x-3)(10) + (x-3)(x-0)(5) + (x-3)(x-0)(x-4)(1)$$
$$= x^3 - 2x^2 + 7x - 5$$

Dari hasil b) tampak bahwa pergantian index i maupun dari hasil c) pergantian susunan data tidak mempengaruhi hasil akhir.

Jadi dalam metoda beda terbagi : pergantian susunan index ataupun data → tidak ada pengaruhnya.

III.2.

i	x_i	$f(x_i)$	$f_1[\,]$	$f_2[\,]$	$f_3[\,]$	$f_4[\,]$
0	2.0	1.414214				
			0.34924			
1	2.1	1.449138		−0.04110		
			0.34102		0.009167	
2	2.2	1.483240		−0.03835		−0.002084
			0.33335		0.008333	
3	2.3	1.516575		−0.03585		
			0.32618			
4	2.4	1.549193				

- *Beda terbagi Newton*

$$p_1(x) = f(x_0) + (x-x_0)\,f[x_0, x_1]$$
$$= 1.414214 + (x-2)\,0.34924$$
$$= 0.34924x + 0.715734$$
$$p_1(2.05) = 0.34924(2.05) + 0.715734 = 1.431676$$
$$p_1(2.15) = 1.4666$$
$$p_1(2.45) = 1.571372$$

$$p_2(x) = p_1(x) + (x-x_0)(x-x_1)\,f[x_0, x_1, x_2]$$
$$= p_1(x) + (x-2)(x-2.1)(-0.0411)$$
$$= p_1(x) - 0.0411\,(x^2 - 4.1x + 4.2)$$
$$p_2(2.05) = p_1(2.05) - 0.0411\,(2.05^2 - 4.1 \times 2.05 + 4.2) = 1.431779$$
$$p_2(2.15) = 1.466292$$
$$p_2(2.45) = 1.564899$$

$$p_3(x) = p_2(x) + (x-x_0)(x-x_1)(x-x_2)\,f[x_0, x_1, x_2, x_3]$$
$$= p_2(x) + (x-2)(x-2.1)(x-2.2)(0.009167)$$
$$p_3(2.05) = p_2(2.05) + (2.05-2)(2.05-2.1)(2.05-2.2)(0.009167)$$
$$= 1.431782$$
$$p_3(2.15) = 1.466288$$
$$p_3(2.45) = 1.565260$$

$$p_4(x) = p_3(x) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\,f[x_0, x_1, x_2, x_3, x_4]$$
$$= p_3(x) + (x-2)(x-2.1)(x-2.2)(x-2.3)(-0.002084)$$
$$p_4(2.05) = 1.431782$$
$$p_4(2.15) = 1.466288$$
$$p_4(2.45) = 1.565247$$

x_i	$f(x_i)$	$f_1[\,]$	$f_2[\,]$	$f_3[\,]$	$f_4[\,]$
2.0	1.414214				
		0.034924			
2.1	1.449138		-0.000822		
		0.034102		0.000055	
2.2	1.483240		-0.000767		-0.000005
		0.033335		0.000050	
2.3	1.516575		-0.000717		
		0.032618			
2.4	1.549193				

- Beda maju. $p_n(x) = f_0 + \mu \Delta f_1 + \dfrac{\mu(\mu-1)}{2!}\Delta f_2 + \cdots$

$$x = 2.05 \rightarrow \mu = \frac{x - x_0}{h} = \frac{2.05 - 2}{0.1} = 0.5$$

$$p_1(2.05) = 1.414214 + 0.5(0.034924) = 1.431676$$

$$p_2(2.05) = p_1(2.05) + \frac{0.5(0.5-1)}{2!}(-0.000822) = 1.431779$$

$$p_3(2.05) = p_2(2.05) + \frac{(0.5)(-0.5)(-1.5)}{3!}(0.000055) = 1.431782$$

$$p_4(2.05) = p_3(2.05) + \frac{0.5(-0.5)(-1.5)(-2.5)}{4!}(-0.000005) = 1.431782$$

$$x = 2.15 \rightarrow \mu = 1.5$$

$$p_1(x) = 1.414214 + 1.5(0.034924) = 1.4666$$

$$p_2(x) = p_1(x) + \frac{1.5(0.5)}{2!}(-0.000822) = 1.466292$$

$$p_3(x) = p_2(x) + \frac{1.5(0.5)(-0.5)}{3!}(0.000055) = 1.466288$$

$$p_4(x) = p_3(x) + \frac{1.5(0.5)(-0.5)(-1.5)}{4!}(-0.000005) = 1.466288$$

$x = 2.45 \rightarrow \mu = 4.5$

$p_1(x) = 1.414214 + 4.5(0.034924) = 1.571372$

$p_2(x) = p_1(x) + \dfrac{4.5(3.5)}{2!}(-0.000822) = 1.564899$

$p_3(x) = p_2(x) + \dfrac{4.5(3.5)(2.5)}{3!}(0.000055) = 1.565260$

$p_4(x) = p_3(x) + \dfrac{4.5(3.5)(2.5)(1.5)}{4!}(-0.000005) = 1.565247$

- Beda mundur : $p_n(x) = f(x_0) + (-\alpha)\nabla f_1 + \dfrac{(-\alpha)(-\alpha+1)}{2!}\nabla f_2 + \cdots$

$x = 2.05 \rightarrow \alpha = \dfrac{x_0 - x}{h} = \dfrac{2.4 - 2.05}{0.1} = 3.5$

$p_1(x) = 1.549193 + (-3.5)(0.032618) = 1.43503$

$p_2(x) = p_1(x) + \dfrac{(-3.5)(-2.5)}{2!}(-0.000717) = 1.431893$

$p_3(x) = p_2(x) + \dfrac{(-3.5)(-2.5)(-1.5)}{3!}(0.00005) = 1.431784$

$p_4(x) = p_3(x) + \dfrac{(-3.5)(-2.5)(-1.5)(-0.5)}{4!}(-0.000005) = 1.431782$

$x = 2.15 \rightarrow \alpha = 2.5$

$p_1(x) = 1.549193 + (-2.5)(0.032618) = 1.467648$

$p_2(x) = p_1(x) + \dfrac{(-2.5)(-1.5)}{2!}(-0.000717) = 1.466304$

$p_3(x) = p_2(x) + \dfrac{(-2.5)(-1.5)(-0.5)}{3!}(0.00005) = 1.466288$

$p_4(x) = p_3(x) + \dfrac{(-2.5)(-1.5)(-0.5)(0.5)}{4!}(-0.000005) = 1.466288$

$x = 2.45 \rightarrow \alpha = -0.5$

$p_1(x) = 1.549193 + 0.5(0.032618) = 1.565502$

$p_2(x) = p_1(x) + \dfrac{(0.5)(1.5)}{2!}(-0.000717) = 1.565233$

$p_3(x) = p_2(x) + \dfrac{(0.5)(1.5)(2.5)}{3!}(0.00005) = 1.565249$

$p_4(x) = p_3(x) + \dfrac{(0.5)(1.5)(2.5)(3.5)}{4!}(-0.000005) = 1.565247$

III.3. Interpolasi Lagrange

$$p_7(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + \cdots + L_7(x) f(x_7)$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_7)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_7)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)\ldots(x-x_7)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_7)}$$

$$\vdots$$

$$L_7(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_6)}{(x_7-x_0)(x_7-x_1)\ldots(x_7-x_6)}$$

Data

i	x_i	$f(x_i)$
0	0	32.0
1	500	176.0
2	1000	296.4
3	2000	509.0
4	3000	704.7
5	4000	891.9
6	5000	1072.6
7	6000	1247.5

$$p_7(300) = 5.661906 + 207.063982 - 134.854463 + 79.464431 - 55.415903 + 27.418366 - 8.075745 + 1.056105 = 122.318679$$

$$p_7(1700) = 0.950389 - 32.82615 + 128.272026 + 428.318741 - 109.476639 + 41.954842 - 10.940398 + 1.331619 = 447.58443$$

$$p_7(3300) = 0.717313 - 20.611768 + 57.197132 - 144.816175 + 695.049359 + 201.968659 - 31.115032 + 3.107111 = 761.496599$$

$$p_7(5900) = 4.548792 - 121.172746 + 304.39074 - 547.294704 + 815.200306 - 843.633561 + 666.349522 + 951.143261 = 1229.531604$$