By equation (21) since $z_1 = z_2$, $p_1 = p_2$, $u_1 = u_2$, $(1 + k)v_2^2 = v_1^2$,

$$v_2 = \frac{v_1}{\sqrt{1 + k}} = \frac{V_1 - u_1}{\sqrt{1 + k}}$$

Substituting this value of v_2 we obtain

$$F = \frac{W}{g}\left(1 - \frac{\cos \beta_2}{\sqrt{1 + k}}\right)(V_1 - u_1) \qquad (28)$$

A more exact value for the force exerted may be found in Art. 72. The above is only an approximation.

Multiplying the force given above by the velocity of its point of application, we have the power developed. Thus

$$P = Fu_1 = \frac{W}{g}\left(1 - \frac{\cos \beta_2}{\sqrt{1 + k}}\right)(V_1 - u_1)u_1 \qquad (29)$$

The power input to the wheel, including the nozzle, is Wh, where h is determined as in Art. 55. The power in the jet is $WV_1^2/2g$ and is less than the former by the amount lost in friction in the nozzle.

Equation (28) is the equation of a straight line. It shows that F is a maximum when u_1 is zero and that it decreases with the speed until it becomes zero when $u_1 = V_1$. Equation (29) is the equation of a parabola. It shows that the power is zero when $u_1 = 0$ and again when $u_1 = V_1$. The vertex of the curve, which gives the maximum power and hence the maximum efficiency, is found when $u_1 = 0.5V_1$. Since in reality both of these curves are altered somewhat, when all the factors are considered, some of these statements require modification.

The actual speed for the highest efficiency has been found by test to be such that $\phi_e = 0.45$ approximately, while the value of the efficiency is about 80 per cent. Both of these values vary somewhat with the design of the wheel and the conditions of use. But one can approximately compute the bucket speed and the power of any impulse wheel, provided the head and size of jet are known. The bucket speed $u_1 = \phi\sqrt{2gh}$, while the velocity of the jet $V_1 = c_v\sqrt{2gh}$. For a good nozzle with full opening, if equipped with a needle, the coefficient of velocity should be about 0.98. Thus the rate of discharge is determined. If the diameter of the wheel is known, or assumed as a function of the size of the jet, the rotative speed can be computed.

The reasons for the modifications of the simple theory given above and an analysis of the characteristics of an actual wheel are given in the following parts of this chapter.

70. The Angle α_1.—The angle α_1 is usually not zero as can be seen from Fig. 83. One bucket will be denoted by B and the bucket just ahead of it by C. Different positions of these buckets will be denoted by suffixes. The bucket enters the jet when it is at B_1 and begins to cut off the water from the preceding bucket C_1. When the bucket reaches the position B_2 the last drop of water will have been cut off from C_2, but there will be left a portion of the jet, $MPXY$, still acting upon it. The last drop of water X will have caught up with this bucket when it reaches position C_3. Thus while the jet has been striking it the bucket has turned

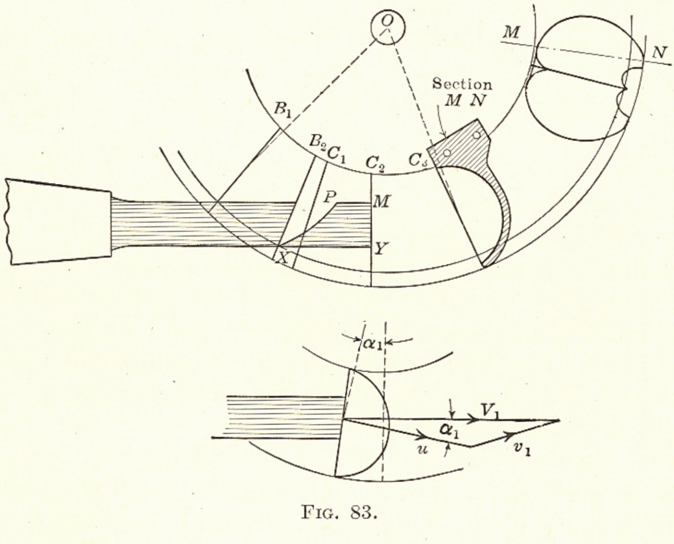
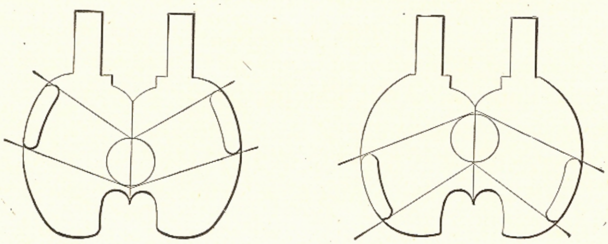


FIG. 83.

through the angle B_1OC_3. The average value of α_1 will be taken as the angle obtained when the bucket occupies the mean between these two extreme positions. It is evident that position C_3 will depend upon the speed of the wheel, and that the faster the wheel goes the farther over will C_3 be. Thus the angle α_1 decreases as the speed of the wheel increases. The variation in the value of α_1 as worked out for one particular case is shown in Fig. 85.

71. The Ratio of the Radii.—It is usually assumed that $r_1 = r_2$. However inspection of the path of the water in Fig. 84 (*a*) will show that when the bucket first enters the jet r_2 may be less than r_1. When the bucket has gotten further along r_2 may be greater than r_1. The value of $x(=r_2/r_1)$ depends upon the design of the buckets, and its determination is a drafting-board problem which is not within the scope of this book. It is evident that a value of x must be a mean in the same way that a value of α_1 is a mean. And just as α_1 varies with the speed, so also does x vary with the speed. A little thought will show that when the wheel is running slowly compared with the jet velocity the value of x will be less than when the wheel is running at a higher speed. This may be verified by actual observation. When the wheel is running at its proper speed it is probably true that x is very nearly equal to unity. In any case the variation of the value of x from unity cannot be very great.



(a) (b)

FIG. 84.—Radii for different bucket positions.

72. Force Exerted.—The force acting on the wheel may be determined by the principles of Art. 58, but, if the radii are not equal it will not be convenient to use equation (15) on account of the difficulty of locating the line of action of the force. However we can use equation (17) and by it determine a force at the radius r_1 which shall be the equivalent of the real force. Dividing (17) by r_1 and letting F denote tangential force we obtain

$$F = \frac{W}{g}(V_{u1} - xV_{u2})$$

$$V_{u1} = V_1 \cos \alpha_1$$

$$V_{u2} = u_2 + v_2 \cos \beta_2.$$

By equation (21)

$$(1 + k)v_2^2 = v_1^2 + u_2^2 - u_1^2.$$

By trigonometry

$$v_1^2 = V_1^2 - 2V_1u_1 \cos \alpha_1 + u_1^2.$$

Substituting this value of v_1, and with $u_2 = xu_1$,

$$(1 + k)v_2^2 = V_1^2 - 2V_1u_1 \cos \alpha_1 + x^2u_1^2.$$

Substituting v_2 from this in the expression for V_{u2} we obtain

$$F = \frac{W}{g}\left[V_1 \cos \alpha_1 - x^2u_1 - \frac{x \cos \beta_2}{\sqrt{1+k}}\sqrt{V_1^2 - 2V_1u_1 \cos \alpha_1 + x^2u_1^2}\right] \quad (30)$$

Equation (30) is a true expression for the force exerted. No great error is involved, however, by taking $x = 1.0$. If that is done the expression under the radical becomes the value of v_1 and may be found graphically. For the sake of simplicity and ease in computation α_1 may be taken equal to zero and the equation then reduces to (28), but an exact value of F will not be obtained. There is little excuse for taking $k = 0$, as most writers do, for equation (28) is not simplified to any extent and the results are entirely incorrect.

73. Power.—With F as obtained from (30) the power is given by Fu_1. We may also compute h'' and obtain the power by multiplying by W.

Since $h'' = \frac{1}{g} u_1 (V_{u1} - xV_{u2})$ it is evident that the expression for h'' is the same as (30) if u_1 be substituted for W. Thus the expression for power has the same value no matter from which basis it is derived.

74. The Value of W.—W is the total weight of water striking the wheel per second. It is obvious that the weight of water discharged from the nozzle is

$$W = w A_1 V_1.$$

Under normal circumstances all of this water acts upon the wheel. However for high values of the ratio u_1/V_1 a certain portion of the water may go clear through without having had time to catch up with the bucket before the latter leaves the field of action. It is apparent, for instance, that if the buckets move as fast as the jet none of the water will strike them at all. For all speeds less than that extreme case a portion of the water only may fail to act. Thus referring to Fig. 83, it can be seen that if the wheel speed is high enough compared to the jet velocity the water at X may not have time to catch up with bucket C. The variation of W with speed is shown in a particular case by Fig. 85.

It may also be seen that the larger the jet compared to the diameter of the wheel the lower the value of u_1/V_1 at which this loss will begin to occur and it is not desired to have it occur until the normal wheel speed is exceeded. Thus there is a limit to the size of jet that may be used for a given wheel, as stated in Art. 30. For a given diameter of wheel, as the size of the nozzle is increased, larger buckets must be used and they must also be spaced closer together.

75. The Value of k.—The value of k is purely empirical and must be determined by experiment. If the dimensions of the wheel are known and the mechanical friction and windage losses are determined or estimated, then from the test of the wheel the horse-power developed by the water may be obtained. The value of k is then the only unknown quantity and may be solved

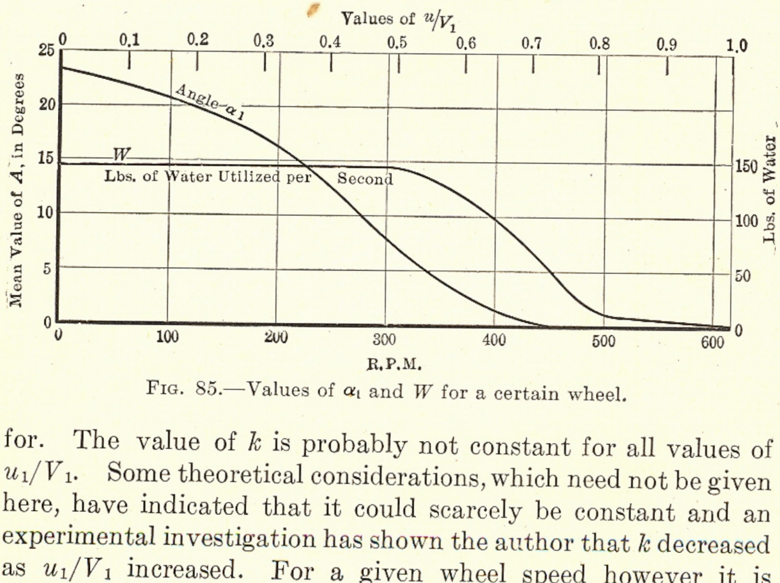


FIG. 85.—Values of α_1 and W for a certain wheel.

for. The value of k is probably not constant for all values of u_1/V_1. Some theoretical considerations, which need not be given here, have indicated that it could scarcely be constant and an experimental investigation has shown the author that k decreased as u_1/V_1 increased. For a given wheel speed however it is nearly constant for various needle settings unless the jet diameter exceeds the limit set in Art. 30. The crowding of the bucket then increases the eddy losses and would require a higher value of k.

The value of k may be as high as 2.0 but the usual range of values is from 0.5 to 1.5.

76. Constant Input—Variable Speed.—The variation of torque and power with speed for different needle settings is shown by Fig. 86 and Fig. 87. With the wheel at rest the torque may vary within certain limits as is shown by the curve for full nozzle opening. This is due to differences in α_1 and in x for various positions of the buckets. When running at a slow speed the brake reading was observed to fluctuate between the limits shown. At higher speeds this could not be detected. This action is here shown for only one nozzle opening but it exists for all. With a given nozzle opening the horsepower output is fixed and constant. The horsepower output varies with the speed. It will be noticed that the maximum efficiency is attained at slightly higher speeds

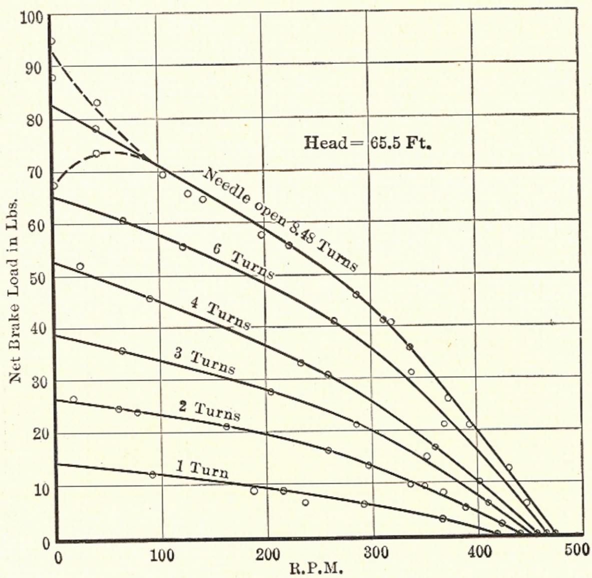


FIG. 86.—Relation between torque and speed.

for the larger nozzle openings than for the smaller. This is due, in part, to the fact that the mechanical losses, which are practically constant at any given speed, become of less relative importance as the power output increases.

Fig. 88 shows the variation of the different losses for a constant power input but a variable speed.[1]

77. Best Speed.—It is usually assumed that the best speed is the one for which the discharge loss is the least. As shown in Art. 64, the latter will be approximately attained either when $u_2 = v_2$ or when $\alpha_2 = 90°$. In the case of the impulse turbine the former

[1] The curves shown in this chapter are from the test of a 24-in. tangential water wheel by F. G. Switzer and the author.

assumption gives an easier solution. It will be found that $u_2 = v_2$ if u_1 is found from

$$kx^2u_1^2 + 2V_1u_1 \cos A_1 - V_1^2 = 0.*$$

An inspection of the curves in Fig. 88 will show that the highest efficiency is not obtained when the discharge loss is the least. So

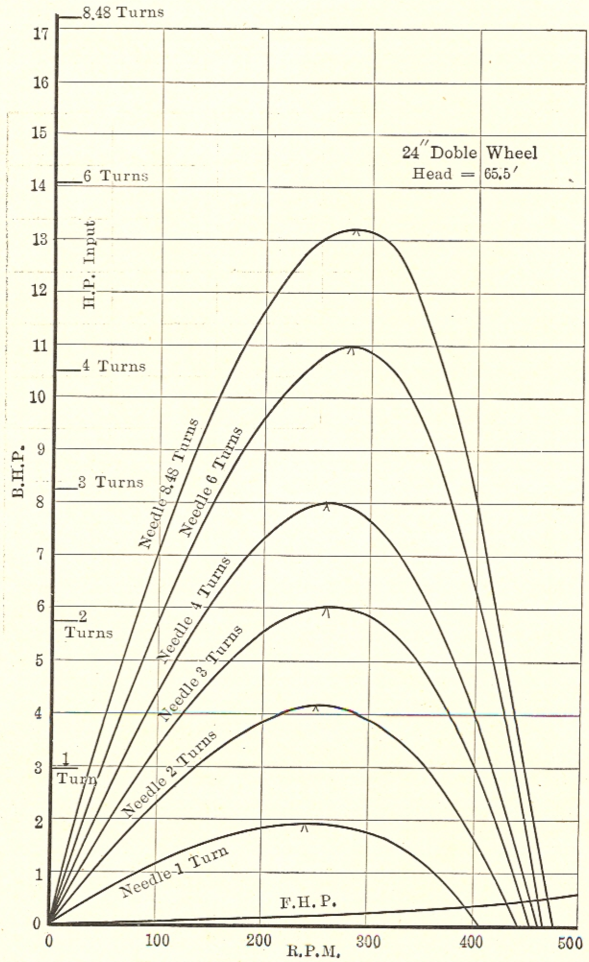


Fig. 87.—Relation between power and speed for different needle settings.

that, although the difference is not great, the above equation does not give the best speed. The hydraulic friction losses and

* L. M. Hoskins, "Hydraulics," Art. 198, Art. 208.

the bearing friction and windage cause the total losses to become a minimum at a slightly higher speed. It does not seem possible to compute this in any simple way but it will be found that the best speed is usually such that $u_1/V_1 = 0.45$ to 0.49.

The speed of any turbine is generally expressed as $u_1 = \phi\sqrt{2gh}$. The coefficient of velocity of the nozzle will reduce the above

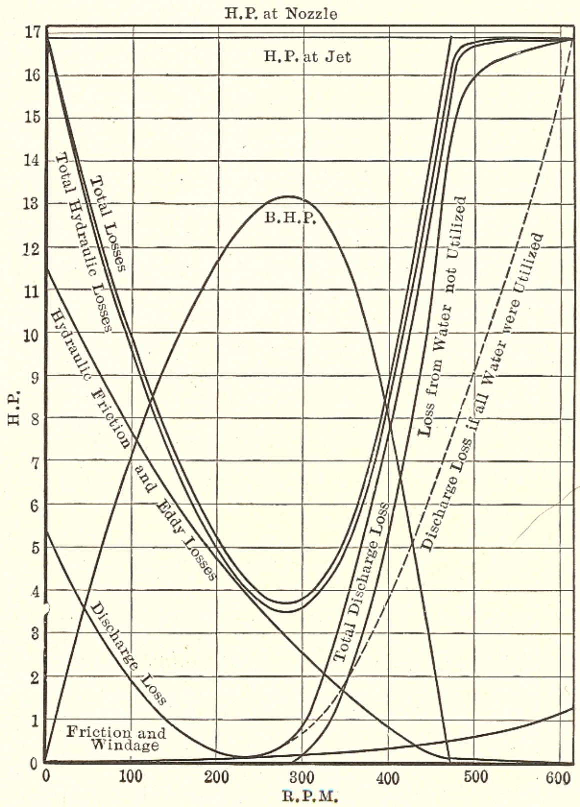


FIG. 88.—Segregation of losses for constant input and variable speed.

values slightly, so that the best speed is usually such that

$$\phi_e = 0.43 \text{ to } 0.47$$

78. Constant Speed—Variable Input.—The case considered in Art. 76 is valuable in showing us the characteristics of the wheel but the practical commercial case is the one where the speed is constant and the input varies with the load. From Fig. 87 it is

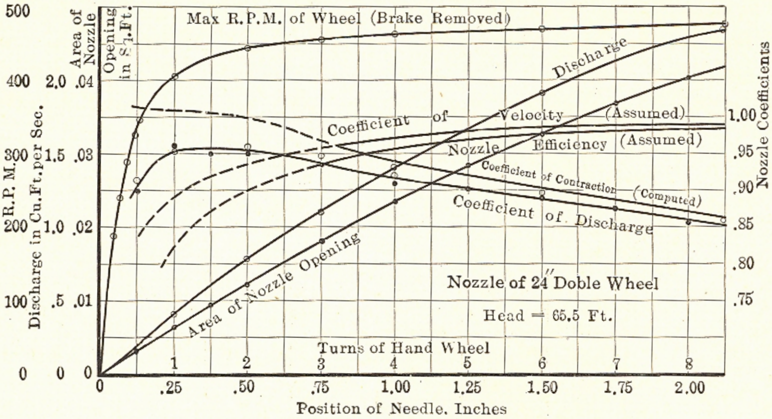


FIG. 89.—Nozzle coefficients and other data.

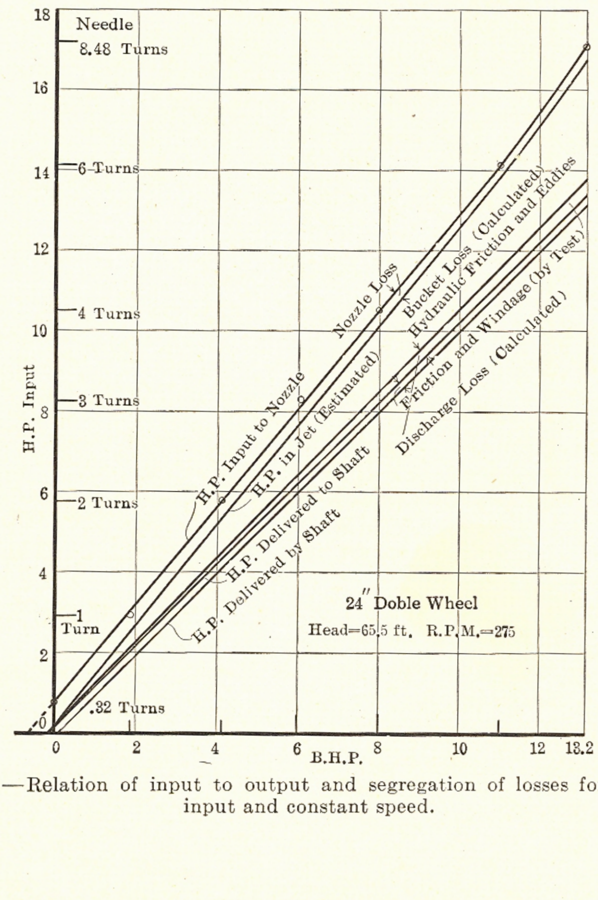


FIG. 90—Relation of input to output and segregation of losses for variable input and constant speed.

seen that the best speed is 275 r.p.m. That value was taken because the highest efficiency was obtained with the nozzle open six turns. For that value of N the curves in Fig. 90 were plotted. It will be noted that the relation between input and output is

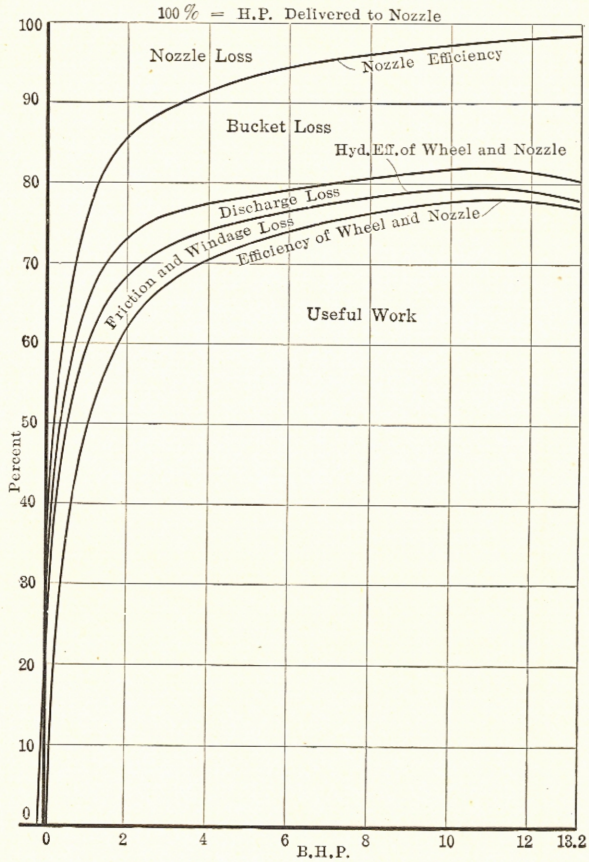


FIG. 91.—Efficiencies and per cent. losses at constant speed.

very nearly a straight line. Above six turns it bends up slightly because the wheel is then slightly overloaded.

The friction and windage was determined by a retardation run[1] and was assumed to be constant at all loads. The hydraulic losses were segregated by the theory already given (Art. 62). These results plotted in per cent. are shown in Fig. 91 and Fig. 92.

[1] See Art. 101.

cause, if only a very few points are determined by test, the complete curve can be drawn with a reasonable degree of accuracy.

The theory also shows that the hydraulic efficiency of the wheel alone is nearly constant from no-load to full-load at constant speed. And considering the efficiency of the wheel and nozzle

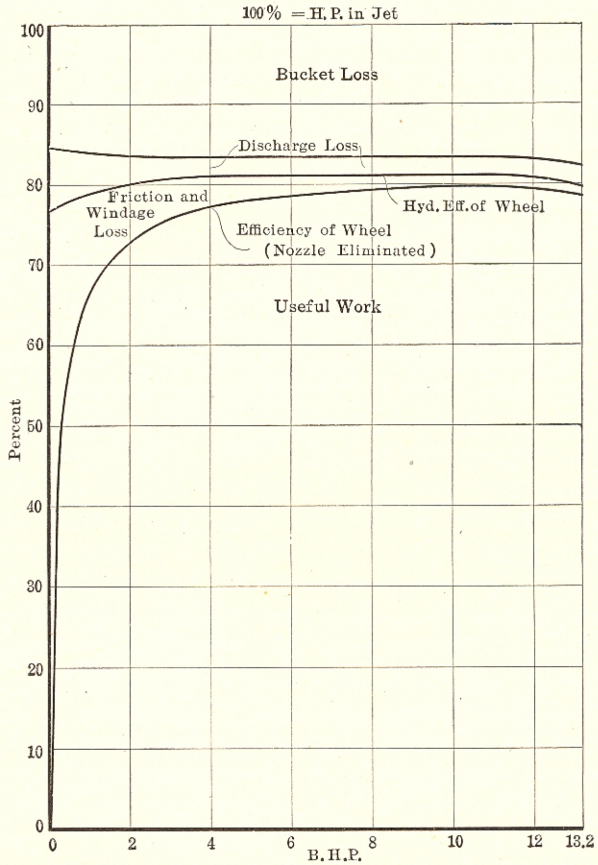


FIG. 92.—Efficiencies and per cent. losses at constant speed based upon power in jet.

together the hydraulic efficiency does not begin to drop off rapidly until very small nozzle openings are reached. The reason for this is that the vector velocity diagrams upon which the theory is based are independent of the size of the jet. The variations shown in Fig. 92 are due to changes in c_v and k. This is of practical importance as showing why impulse wheels have relatively flat efficiency curves.

80. Illustrative Problem.—Referring to Fig. 93 let the total fall to the mouth of the nozzle be 1000 ft. Suppose BC = 5000 ft. of 30-in. riveted steel pipe and at C a nozzle be placed whose coefficient of velocity = 0.97. Suppose the diameter of the jet from the nozzle = 6 in. Let this jet act upon a tangential water wheel of the following dimensions: Diameter = 6 ft., $\alpha_1 = 12°$, $\beta_2 = 170°$. Assume $k = 0.6$, $\phi = 0.465$, and assume bearing friction and windage = 3 per cent. of power input to shaft.

The problem of the pipe line is a matter of elementary hydraulics and a detailed explanation will not be given of the steps here employed. The coefficient of loss at B will be taken as

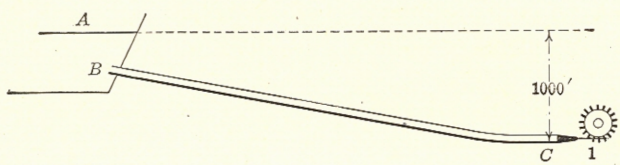


FIG. 93.

1.0, the coefficient of loss in the pipe will be assumed 0.03. The loss in the nozzle will be given by $\left(\frac{1}{c_v^2} - 1\right)\frac{V_1^2}{2g}$, where c_v = the coefficient of velocity and V_1 the velocity of the jet. If V_c = the velocity in the pipe then the losses will be

$$\left(1 + 0.03\frac{5000}{2.5}\right)\frac{V_c^2}{2g} + 0.063\frac{V_1^2}{2g}$$

Taking $H_A = 1000$ ft. and $H_1 = \frac{V_1^2}{2g}$ then by equation (4) we may solve for $\frac{V_c^2}{2g} = 1.38$ ft. or $\frac{V_1^2}{2g} = 861$ ft.

Thus $V_c = 9.42$ ft. per second and $V_1 = 235.5$ ft. per second.

Rate of discharge, $q = 4.62$ cu. ft. per second.

The pressure head at nozzle, $\frac{p_c}{w} = 914.5$ ft.

The wheel speed $u_1 = 0.465 \times 8.025\sqrt{915.88} = 113$ ft. per second.

Therefore $N = 360$ r.p.m.

By methods illustrated in Art. 62, $v_1 = 126.7$ ft. per second, $v_2 = 100$ ft. per second, and $V_{u2} = 14.5$, assuming $x = 1.0$. Thus, $h'' = \frac{u_1}{g}\left(V_{u1} - V_{u2}\right) = 757$ ft.

The means of obtaining the following answers will doubtless be obvious.

Total head available,	$H_A = 1000$ ft.
Head at nozzle,	$H_c = 915.88$ ft.
Head in jet,	$H_1 = 861$ ft.
Head utilized by wheel,	$h'' = 757$ ft.

Total power available at A	= 5250 h.p.
Power at nozzle (C)	= 4800 h.p.
Power in jet	= 4520 h.p.
Power input to shaft	= 3970 h.p.
Power output of wheel	= 3851 h.p

Hydraulic efficiency of wheel	= 0.878
Mechanical efficiency of wheel	= 0.970
Gross efficiency of wheel	= 0.852
Efficiency of nozzle	= 0.941
Gross efficiency of wheel and nozzle	= 0.801
Efficiency of pipe line BC	= 0.915
Overall efficiency of plant	= 0.733

81. QUESTIONS AND PROBLEMS

1. With the simple theory of the tangential wheel what are the relations for torque and power as functions of speed? How may the speed and power of an impulse wheel be computed in practice?

2. What are the true conditions of flow in the Pelton water wheel and what assumptions are often made in order to simplify the theory?

3. When may a portion of the water discharged from a nozzle fail to act upon the wheel? Why? What changes in design will improve this condition?

4. Why is the relation between input and output at a constant speed and head not a straight line throughout its range? How does the hydraulic efficiency vary from no-load to full-load at constant speed? Why?

5. Suppose the dimensions of a tangential water wheel are: $\beta_2 = 165°$, $\phi = 0.45$, $k = 0.5$, and the velocity coefficient of the nozzle = 0.98. If the diameter of the jet = 8 in. and the head on the nozzle 900 ft., compute the value of the force exerted on the wheel, assuming $\alpha_1 = 0°$ and $x = 1.0$.

6. Compute the force on the wheel in problem (5) assuming $\alpha_1 = 20°$.

7. Compute the hydraulic efficiency of the wheel in problem (5). Is this dependent upon the value of the head?

8. Derive an equation for the hydraulic efficiency of a Pelton wheel, giving the result in terms of wheel dimensions and factors such as ϕ and c_v.

9. Suppose it is desired to develop 2000 h.p. at a head of 600 ft. Assuming an efficiency of 600 ft., what will be the size of jet required, and what will be the approximate diameter and r.p.m. of the wheel?