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AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE HYDRAULIC RAM

BY

WALLACE M. LANSFORD

AND

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AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE HYDRAULIC RAM

I. INTRODUCTION

1. *Preliminary Statement.*—The hydraulic ram is a machine which uses the kinetic energy in a moving column of water to lift part of that water to a height greater than that of the source of supply. With no external addition of energy, a well-made hydraulic ram will pump a part of the water with which it is supplied to any height from about twice the supply head to many times the supply head, with an efficiency which exceeds 65 per cent or even 70 per cent in some ranges of capacity and pressure.

A patent on the hydraulic ram was issued in 1797, to Montgolfier[1]* in France. Since that time, its dependability, simplicity, and economy have led to its frequent use, particularly in isolated districts where no water mains are available. Its application varies from individual installations supplying families or livestock, to larger systems supplying water to small cities.

Despite the widespread use of the ram, the analysis of its actual operating characteristics is not generally known, though empirical performance guarantees are of course made by manufacturers. Although several investigators have studied the problem of a rational analysis, only O'Brien and Gosline[2] may be said to have arrived at a solution. Their solution applies only to rams having a very rigid or stiff waste valve.

2. *Purpose of Investigation.*—It was the purpose of the present investigation to make a rational mathematical analysis of the operation of single-acting automatic hydraulic rams, and to compare the results of such an analysis with those found from an experimental investigation of the rams.

The rams used in this investigation differed considerably in construction and performance from the one used by O'Brien and Gosline. The analysis developed by O'Brien and Gosline neglected the effects of elasticity of the drive pipe and waste valve parts, while in the rams used in this investigation, the effects of elasticity of the resilient waste valve disc were often more marked than the effects of the elasticity of the water. These differences in construction and performance seemed to justify further investigation.

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*Numerical indices refer to correspondingly numbered references in the bibliography at the end of the bulletin.

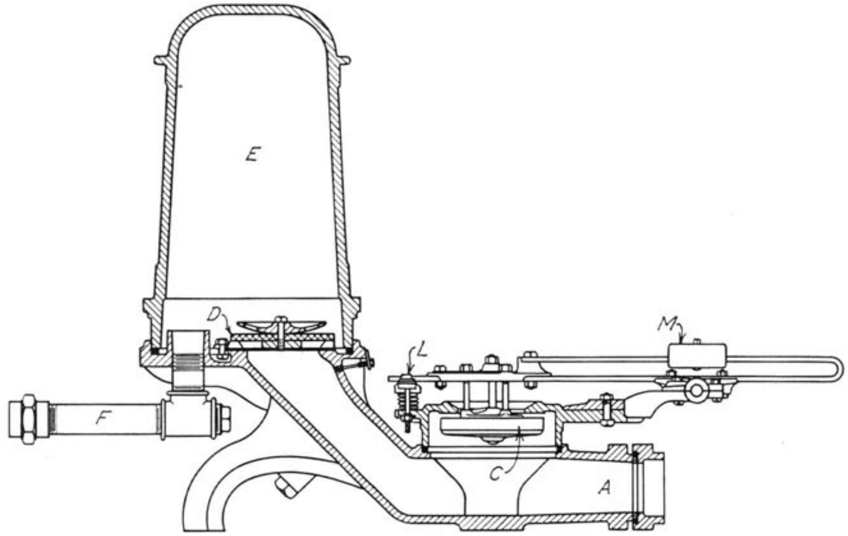

FIG. 1. CROSS-SECTIONAL DIAGRAM OF A HYDRAULIC RAM

3. *Acknowledgment.*—The investigation herein presented was started as a thesis by the second-named author under the supervision of the first-named author. The investigation was continued as a part of the work of the Engineering Experiment Station, of which DEAN M. L. ENGER is the director, and of the Department of Theoretical and Applied Mechanics, of which PROFESSOR F. B. SEELY is the head.

During the investigation pressure–time diagrams for the ram were obtained by use of an electrical pressure gage and an oscillograph which was adapted for use with the ram and operated by the staff of the "Stresses in Railroad Track Investigation." The authors wish to express their appreciation especially to Mr. R. Ferguson, Mr. H. C. Roberts, and Mr. T. P. DeWan of this investigation for the interest shown and help given in operating the apparatus as applied to the ram.

4. *Explanation of Operation of Ram.*—In Fig. 1 is shown a diagram of the valve box of a hydraulic ram manufactured by the Rife Hydraulic Manufacturing Company which will help make clear the operation of the ram. At *A* is attached the drive pipe, the other end of which is connected to the water supply. If the waste valve *C* is open, as shown in the diagram, water flows into the valve box, then out around the waste valve disc *C*, and is wasted.

As the flow increases, a frictional resistance is developed in the waste valve passage which increases the pressure under the valve disc C. When the pressure in the valve box is large enough, the valve and rocker-arm on which it is mounted rise, thereby closing the valve quickly. Thus the flow through the waste valve is stopped, but since the column of water in the drive pipe has a considerable velocity, a very high pressure would be created in the valve box if relief were not provided. The relief is found in the opening of the check valve D, which permits the flow of the water to continue by passing into the surge tank E; this tank is filled partly with water and partly with air. The pipe at F is connected to the delivery tank, so that the pressure in the surge tank E is the delivery pressure. Water continues to flow into the surge tank until the higher pressure which exists there reduces the velocity to zero. The check valve then closes and the waste valve opens, completing the cycle. The duration of the cycle is usually from one-half to one and one-half seconds. The surge tank is large enough to cause the flow from it to be sensibly steady, though the pumping into it is intermittent.

5. *Objectives of Analysis.*—Many variables are involved in the operation just described, but the ones of primary interest to the designing or operating engineer are the rate of pumping and the rate of wasting (weight of water pumped or wasted per unit of time). Therefore the objective of the analysis herein presented was to determine the rate of pumping and of wasting for any condition of operation, having previously obtained the physical dimensions and the experimental constants of the apparatus for that condition of operation. A list of constants which are necessary for the analysis is given in the last column in Table 1 (page 36). The definitions of the symbols for the constants are given on page 12.

II. ANALYSIS OF ACTION OF RAM

6. *Method of Approach.*—The problem is attacked by attempting to obtain the relation between velocity and time for the water column in the drive pipe during each part of the cycle. From this relationship the quantity of water wasted and the quantity pumped per cycle may be found; the time elapsed during a cycle may also be determined. From these values the time rate of pumping and of wasting may be easily determined.

7. *Division of Cycle into Periods.*—It was necessary to divide the cycle into its separate parts rather precisely, and to analyze each

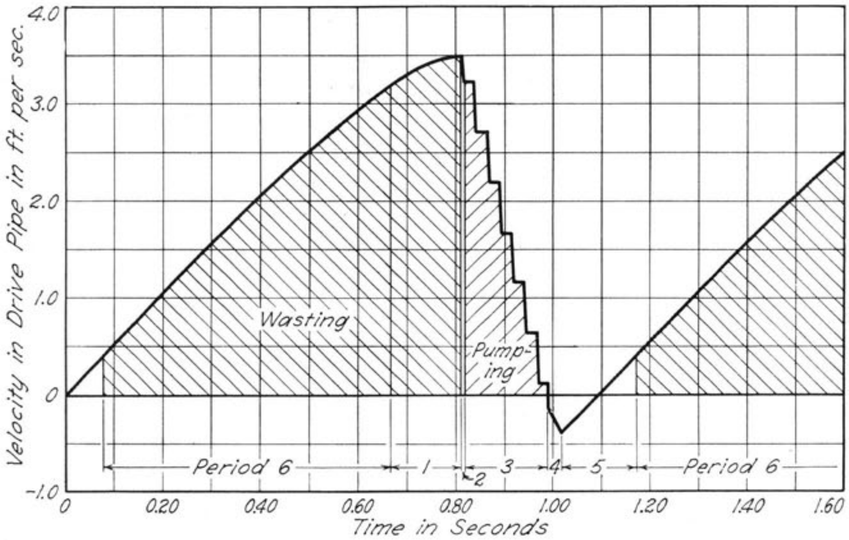

FIG. 2. RELATION OF VELOCITY IN DRIVE PIPE AT RAM TO TIME

part individually. It seemed logical to divide the cycle into six separate periods, during each of which the velocity is affected by different factors and therefore varies according to a different law. Reference to Fig. 2, which is a diagram of the assumed relation of velocity in the drive pipe to time, will clarify the six periods. In the analysis of the ram the drive pipe was assumed to be horizontal.

The cycle was considered to start at the instant of the beginning of waste valve closure. This was a logical point in the cycle at which to begin the analysis, since, from a consideration of the factors controling the operation of the waste valve, it seemed reasonable to assume that the velocity of the water in the drive pipe at this instant is independent of supply pressure and delivery pressure, and depends only on the setting of the waste valve. At this time water is being wasted through the open waste valve, and the wasting continues until the valve is completely closed. The first period includes the time from the instant the waste valve begins its motion to the instant at which it is completely closed. That is, Period 1 includes the time during which the waste valve closes.

Immediately after the waste valve closes, the pressure in the valve box is only slightly above atmospheric, but the water, traveling at a considerable velocity, compresses itself, expands the pipe, and compresses the rubber waste valve disc, as the pressure rises. When the pressure in the valve box becomes equal to the delivery pressure or slightly greater, the check valve opens. The distance through which the check valve moves is comparatively small and for convenience the motion of the check valve is assumed to be instantaneous. The second period includes the time between the instant of complete closure of the waste valve and the instant of opening of the check valve.

Flow into the surge tank now occurs, and continues until the unbalanced force caused by the difference between delivery and supply pressures reduces the velocity through the check valve to zero. The third period includes the time during which the check valve is open.

The check valve now closes, and the water, under delivery pressure at the valve box, but under supply pressure at the other end of the drive pipe, is accelerated away from the ram, toward the supply, acquiring a "negative" velocity. The displacement due to this negative velocity lowers the pressure in the valve box until the waste valve will no longer remain closed. The fourth period includes the time between the closing of the check valve and the beginning of opening of the waste valve.

The waste valve now being open, the pressure in the valve box is atmospheric, while at the other end of the drive pipe it is still the supply pressure, assuming a horizontal drive pipe. However, the negative velocity, toward the supply, still persists, partially emptying the valve box, until the force caused by the supply pressure retards the velocity of the water column, stops the motion, and finally gives the water column a velocity directed back toward the ram, refilling the valve box. The fifth period is chosen to include the time between the beginning of opening of the waste valve, and the instant the valve box is refilled and wasting begins.

Water continues to waste through the open waste valve until the difference in pressure on the two sides of the valve is enough to overcome its weight, when the waste valve begins to close. The sixth period includes the time from the beginning of wasting to the beginning of waste valve closure. This completes the cycle.

The foregoing division of the cycle into periods may be briefly summarized as follows:

Period 1 includes the time from the instant the waste valve begins to close until the instant at which it is completely closed.

Period 2 includes the time between the instant of complete closure of the waste valve and the instant of opening of the check valve.

Period 3 includes the time during which the check valve is open.

Period 4 includes the time between the closing of the check valve and the beginning of opening of the waste valve.

Period 5 includes the time between the beginning of opening of the waste valve and the instant when wasting begins.

Period 6 includes the time from the beginning of wasting to the beginning of waste valve closure.

This separation into periods is the basis for the analysis of the operation of the ram.

8. *Definition of Notation.*—

Symbol	Definition	Dimensions
A =	Cross-sectional area of drive pipe	ft.^{2}
A_v =	Area of waste valve disc	ft.^{2}
a =	Velocity of pressure–wave transmission in drive pipe	ft./sec.
B =	Area on velocity–time diagram of Fig. 4	ft.
B_r =	Area on velocity–time diagram of Fig. 6	ft.
b =	Friction coefficient of drive pipe	dimensionless
c =	Friction coefficient of waste valve	dimensionless
c_1 =	Constant of integration	
c_2 =	Constant of integration	
E =	Modulus of elasticity of drive pipe material	lb./ft.^{2}
E_v =	Average stiffness modulus of waste valve disc	lb./ft.
e =	Base of natural logarithms, = 2.718	dimensionless
g =	Acceleration of gravity	ft./sec.^{2}
H =	Static supply head	ft.
h =	Static delivery head	ft.
h' =	Instantaneous head in valve box	ft.
h_0' =	Magnitude of pressure wave developed during Period 2	ft.
h_0'' =	Magnitude of pressure wave developed during second surge of Period 3	ft.
h_0 =	Average magnitude of pressure waves developed during Periods 2 and 3	ft.
J =	Acceleration of waste valve during Period 1, assumed constant	ft./sec.^{2}
j =	Combined friction constant, $= b + c + 1$	dimensionless
K =	Volume modulus of elasticity of water	lb./ft.^{2}
k =	Time rate of decrease of acceleration of water column in drive pipe during Period 1, assumed constant	ft./sec.^{3}
L =	Length of drive pipe, supply tank to center of waste valve	ft.
L_1 =	Length of drive pipe, supply tank to center of check valve	ft.
l =	Length of water column in drive pipe	ft.

Symbol	Definition	Dimensions
m =	Frictional constant of check valve, so that head loss $= \frac{mv}{2g}$	ft./sec.
N =	Number of surges in Periods 2 and 3	dimensionless
p =	Functional symbol indicating presence of physical constants in a function	
Q =	Rate at which water is wasted	lb./min.
Q_1 =	Water wasted during Period 1	lb./cycle
Q_6 =	Water wasted during Period 6	lb./cycle
Q_s =	Total water wasted	lb./cycle
q =	Rate at which water is pumped	lb./min.
q_s =	Water pumped	lb./cycle
r =	Radius of drive pipe	ft.
S_0 =	Length of stroke of waste valve	ft.
s =	Value of the displacement of the waste valve from its closed position	ft.
t =	Time	sec.
t_1 =	Duration of Period 1	sec.
t_2 =	Duration of Period 2	sec.
t_3 =	Duration of Period 3	sec.
t_4 =	Duration of Period 4	sec.
t_5 =	Duration of Period 5	sec.
t_6 =	Duration of Period 6	sec.
t_s =	Duration of one cycle	sec.
$t' = \frac{2L_1}{a} - t_2$		sec.
t_r =	Duration of last portion of Period 3, see Fig. 3	sec.
u =	Thickness of wall of drive pipe	ft.
v =	Velocity of water in drive pipe	ft./sec.
v_0 =	Value of v when waste valve begins to close	ft./sec.
v_1 =	Value of v at end of Period 1	ft./sec.
v_2 =	Value of v at end of Period 2	ft./sec.
v_3 =	Value of v at end of Period 3	ft./sec.
v_4 =	Value of v at end of Period 4	ft./sec.
v_5 =	Value of v at end of Period 5	ft./sec.
v_6 =	Value of v at end of Period 6, $= v_0$, since cycle is repeated	ft./sec.
v_r =	Value of v near end of Period 3, see Fig. 3	ft./sec.
Δv =	Reduction in velocity required to produce pressure head h_0	ft./sec.
$(\Delta v)_1$ =	Reduction in velocity required to produce pressure head h_0'	ft./sec.
$(\Delta v)_2$ =	Reduction in velocity required to produce pressure head h_0''	ft./sec.
w =	Specific weight of water	lb./ft.^{3}

Symbol	Definition	Dimensions
Y	= Ratio of change of pressure head in valve box to volume change of valve box $= \dfrac{E_v}{wA_v^2}$	1/ft.^{2}
Z	$= \dfrac{a}{AgY}$	sec.
α_6	= Acceleration of water column in drive pipe at end of Period 6 or beginning of Period 1	ft./sec.^{2}
σ	= Poisson's ratio for drive pipe material	dimensionless

9. *Analysis of Period 1.*—Attempts at rational analysis of the action during this period were fruitless. Some approximations were therefore made.

Records of the actual motion of the waste valve were made during many of the pumping tests, as described later in Section 17. A study of these records showed that the acceleration of the waste valve may be approximated by an equation of the form

$$\frac{d^2s}{dt^2} = -J \tag{1}$$

where s is the value at any instant of the displacement of the waste valve from its closed position, and J is the acceleration of the waste valve, which is assumed constant. Since s is measured away from the valve seat, the negative sign must be used to obtain a positive value for J. The evaluation of the quantity J is described in Section 17.

Integration of Equation (1) gives

$$\frac{ds}{dt} = -Jt + c_1 \tag{2}$$

in which c_1 is a constant of integration. But since the valve starts from rest, $\dfrac{ds}{dt} = 0$ when $t = 0$; hence $c_1 = 0$.

Integration of Equation (2) gives

$$s = -\frac{Jt^2}{2} + c_2. \tag{3}$$

But $s = S_0$ when $t = 0$; hence $c_2 = S_0$. Also $s = 0$ when $t = t_1$. Therefore

$$t_1 = \sqrt{\frac{2S_0}{J}}. \tag{4}$$

An approximation to the variation of velocity in the drive pipe is also necessary. It appears from the indicated pressure–time diagrams, shown in Fig. 16 and described in detail in Section 18, that the pressure in the valve box rises very little until the waste valve is almost completely closed; therefore the velocity of the water column in the drive pipe must continue to increase to some extent during Period 1. On the other hand, the friction through the waste valve undoubtedly increases as the valve opening becomes smaller, and the acceleration of the water column in the drive pipe must decrease due to the increased friction loss and to the force required to accelerate the waste valve. The assumption is made that the acceleration decreases at a constant rate, k, with respect to time, and becomes zero at the instant of valve closure. That is,

$$\frac{d^2v}{dt^2} = -k. \tag{5}$$

Integration of Equation (5) gives

$$\frac{dv}{dt} = -kt + c_1.$$

But $\dfrac{dv}{dt} = \alpha_6$ when $t = 0$; hence $c_1 = \alpha_6$. Also $\dfrac{dv}{dt} = 0$ when $t = t_1$; hence $k = \dfrac{\alpha_6}{t_1}$. Thus,

$$\frac{dv}{dt} = \alpha_6 - \frac{\alpha_6 t}{t_1}. \tag{6}$$

Integration of Equation (6) gives

$$v = \alpha_6 t - \frac{\alpha_6 t^2}{2t_1} + c_2. \tag{7}$$

But $v = v_0$ when $t = 0$; hence $c_2 = v_0$. Also $v = v_1$ when $t = t_1$.

$$\text{Then } v_1 = v_0 + \frac{\alpha_6 t_1}{2}. \tag{8}$$

The velocity, v_0, the minimum velocity at which the waste valve will begin to close, is a function of the shape of valve and valve box, the length of the stroke, and the weight of the waste valve. These factors are not easily rationalized. Therefore, v_0 has been determined experimentally.

Since the value of v_0 is known for any setting of the waste valve, and α_6 and t_1 may be determined from Equation (57) and Equation (4) respectively, v_1 may be calculated by means of Equation (8).

The water wasted during Period 1 is

$$Q_1 = \int_0^{t_1} wAvdt. \tag{9}$$

Substitution of the value of v from Equation (7) gives

$$Q_1 = \int_0^{t_1} Aw\left(v_0 + \alpha_6 t - \frac{\alpha_6 t^2}{2t_1}\right) dt = wA\left(v_0 t_1 + \frac{\alpha_6 t_1^2}{3}\right). \tag{10}$$

From Equation (10) Q_1, the water wasted during Period 1, may be calculated.

At the end of Period 1, then, the water column in the drive pipe is traveling at the known velocity v_1, and has wasted an amount Q_1 in a time t_1. Both waste valve and check valve are closed. Since the acceleration is assumed zero, and pipe friction disregarded, the pressure head is $H - v_1^2/2g$ at all points in the pipe. The velocity head is small compared with the supply head (from 0.4 per cent to 3.7 per cent of the supply head for the tests herein reported), and is even smaller compared with the delivery head. Therefore the velocity head is disregarded in the analysis which follows. That is, the pressure head at all points in the drive pipe is assumed to be H at the end of Period 1.

10. *Analysis of Period 2.*—Continued flow in the drive pipe compresses the waste valve disc, expands the pipe, and compresses the water; these displacements accompany a rise of pressure. Ac-

cording to fundamental water hammer theory, this pressure rise is transmitted back along the drive pipe at the velocity of pressure wave propagation, denoted by a. Further, the velocity of that part of the water column in which the pressure has risen is reduced in direct proportion to the height above H of the pressure head at that instant.[3] That is,

$$\text{reduction in velocity} = \frac{(h' - H)\,g}{a} \tag{11}$$

and if v be now considered as the velocity of the water column at the check valve

$$v = v_1 - \frac{(h' - H)\,g}{a}. \tag{12}$$

The assumption was made that the load–deflection curve for the waste valve disc (see Fig. 7, assumption I) is a straight line, having a slope E_v which is defined as the modulus of the disc. The justification of this assumption is discussed in Section 12. If this assumption is true, a linear relation exists between the change of pressure head in the valve box and the change in volume of the valve box due to the compression of the waste valve disc. That is,

$$\text{change in volume of valve box} = \frac{\Delta h'}{Y}$$

where

$$Y = \frac{\text{Pressure head change in valve box}}{\text{Volume change of valve box}} = \frac{\Delta h'}{\dfrac{A_v w \Delta h'}{E_v} A_v} = \frac{E_v}{w A_v^2}. \tag{13}$$

The change in volume of the valve box must equal the flow into it; that is, for a differential time dt,

$$\frac{dh'}{Y} = A v dt. \tag{14}$$

At the end of Period 2, $v = v_2$, and $(h' - H) = h_0'$, where h_0' is defined as the difference between the head required to force water through the check valve at velocity v_2 and the supply head. Substitution of these values in Equation (12) gives

$$v_2 = v_1 - \frac{h_0'g}{a}. \tag{15}$$

From Equation (15) v_2, the velocity at the end of Period 2, may be computed.

Differentiation of Equation (12) gives

$$dh' = -\frac{a}{g}\,dv. \tag{16}$$

Substitution for dh' from Equation (14) gives

$$AvY\,dt = -\frac{a}{g}\,dv$$

whence

$$dt = -\frac{a}{AgY}\,\frac{dv}{v} = -Z\,\frac{dv}{v} \tag{17}$$

where

$$Z = \frac{a}{AgY} = \frac{awA_v^2}{AgE_v}. \tag{18}$$

Integration of Equation (17) gives

$$t = -Z\log_e v + c_1.$$

But when $t = 0$, $v = v_1$; hence $c_1 = Z\log_e v_1$. Therefore

$$t = Z\log_e\frac{v_1}{v}. \tag{19}$$

This relation of velocity to time continues until the pressure rises enough to open the check valve; at this time the velocity is v_2. Hence the duration of Period 2 is

$$t_2 = Z\log_e\frac{v_1}{v_2}. \tag{20}$$

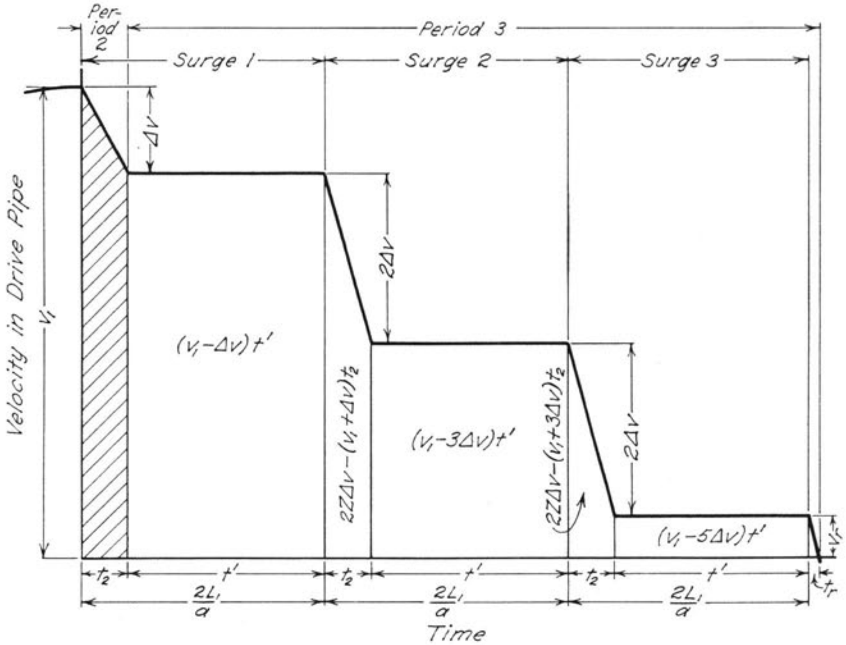

FIG. 3. RELATION OF VELOCITY IN DRIVE PIPE AT RAM TO TIME DURING PERIODS 2 AND 3

It will be noted that the analysis disregards pipe friction during this period. The pressure changes due to pipe friction in a well-installed ram are not important compared to the other pressure variations in this period.

At the end of Period 2, then, the pressure head in the valve box is $h_0' + H$, and the velocity at the check valve is v_2. Beyond a section $a \times t_2$ feet from the check valve measured along the drive pipe toward the supply end, the pressure head H, and velocity v_1, are unchanged from their values at the beginning of Period 2. From the valve box to this section the velocity increases and the rise in pressure over the pressure at the beginning of Period 2 decreases, according to a logarithmic relation.

11. *Analysis of Period 3.*—The analysis of Period 3 extends the work of O'Brien and Gosline to take into account the effects of the elasticity of the waste valve. Reference to Fig. 3, which is a diagram of the assumed relation of velocity at the check valve to time during Periods 2 and 3 will clarify the analysis of these periods. At the beginning of Period 3 the check valve opens, and under the con-

stant differential of pressure head $(h_0' + H) - h$, water is discharged into the surge tank, at a velocity $v_2 = v_1 - (\Delta v)_1$ according to Equation (15) with $(\Delta v)_1$ substituted for $\frac{h_0'g}{a}$. However, in the drive pipe on the supply side of the pressure wave, the velocity is still v_1. As the pressure wave passes any section, the velocity at that section is reduced to $v_1 - (\Delta v)_1$, and the pressure head increased by h_0'. The time required for this pressure rise and velocity decrease to take place at any section is necessarily equal to t_2.

When the first portion of the high-pressure wave reaches the supply end of the drive pipe, the velocity in the drive pipe is $v_1 - (\Delta v)_1$, and the pressure head has been correspondingly increased by an amount h_0', except in that part of the drive pipe at the supply end (of length $a \times t_2$) in which the pressure is still rising. But this higher pressure cannot exist in contact with the constant-pressure supply. Therefore, as the pressure wave reaches the supply end of the pipe, it is reflected back as a "relief" wave, identical in form with the first wave, which reduces the pressure head again by h_0' to its original value H. However, such relief of pressure must reduce the velocity again by $(\Delta v)_1$. Therefore, while the relief wave is traveling back to the ram, the velocity of the water column on the supply side of the wave is $v_1 - 2(\Delta v)_1$ while at the check valve it is still $v_1 - (\Delta v)_1$.

At a time $\frac{2L_1}{a}$ after the beginning of Period 2, the first portion of the relief wave reaches the check valve again, lowering the pressure. The velocity of the water column is still toward the ram, however, and discharge must therefore continue through the check valve. Thus during a length of time t_2, as the first pressure wave is dissipated, another is generated, of magnitude h_0'' which is slightly lower than h_0' because of the reduced friction loss through the check valve at the lower velocity. The generation of the new pressure wave reduces the velocity to $v_1 - 2(\Delta v)_1 - (\Delta v)_2$ in which $(\Delta v)_2$ is less than $(\Delta v)_1$ in the same proportion as h_0'' is less than h_0', as may be seen from Equation (11), by successively substituting $(\Delta v)_1$ and $(\Delta v)_2$ for the left hand member and h_0' and h_0'' for the quantity $(h' - H)$.

The new pressure wave or surge travels back to the supply end of the drive pipe, flow through the check valve meanwhile taking place at the rate of $v_1 - 2(\Delta v)_1 - (\Delta v)_2$. Upon reflection of the second pressure wave as a relief wave, the velocity of the water on the supply side of the wave becomes $v_1 - 2(\Delta v)_1 - 2(\Delta v)_2$, while on the ram side of the wave the velocity is still $v_1 - 2(\Delta v)_1 - (\Delta v)_2$.

This action of one pressure wave succeeding another continues until at last the velocity which remains after dissipation of a pressure wave is insufficient to build up the next pressure wave to a value equal to the delivery pressure. Then the pumping ceases, and Period 3 is completed.

For the sake of convenience in the mathematical treatment of the action in Period 3, the following simplification is made. From Equation (15) and the definition of $(\Delta v)_1$ it is seen that

$$v_1 - v_2 = (\Delta v)_1 = \frac{h_0' g}{a}.$$

The rise in pressure head h_0' necessary to open the check valve is slightly greater than $(h - H)$, due to the friction through the check valve. Therefore h_0' is the sum of the increases in pressure head due to the delivery pressure and the friction loss. That is,

$$h_0' = h - H + \frac{m\,[v_1 - (\Delta v)_1]}{2g} \tag{21}$$

and

$$(\Delta v)_1 = \frac{g}{a}\left\{(h - H) + \frac{m\,[v_1 - (\Delta v)_1]}{2g}\right\}.$$

Subsequent pressure surges in Period 3 are consecutively less in magnitude, since after each surge the velocity through the check valve is less than it was during the preceding surge. When Period 3 is nearly completed, only a small velocity (see Fig. 3) remains; the friction loss through the check valve is small, and therefore the last pressure surge is not much greater than $h - H$, and the velocity reduction due to this pressure surge is nearly equal to $(h - H)\,\dfrac{g}{a}$. For simplicity, the assumption is made that all pressure surges and velocity reductions from beginning to end of Period 3 are equal in magnitude; this magnitude h_0 is taken as the average between the first surge, h_0', and the approximate value of the last surge, $h - H$. Thus

$$h_0 = (h - H) + \frac{m\,(v_1 - \Delta v)}{4g}.$$

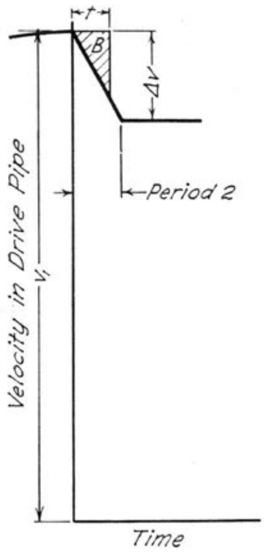

FIG. 4. RELATION OF VELOCITY IN DRIVE PIPE AT RAM TO TIME DURING PERIOD 2

Correspondingly,

$$(\Delta v)_1 = (\Delta v)_2 = \ .\ .\ .\ . = \Delta v.$$

where

$$\Delta v = \frac{h_0 g}{a} = \left[(h - H) + \frac{m\,(v_1 - \Delta v)}{4g}\right]\frac{g}{a}. \tag{22}$$

Such an average introduces no serious error, for when h is high, the friction loss through the check valve is an insignificant part of the pressure variation, and when h is low, there are a large number of surges.

When this assumption is used, Equation (15) becomes

$$v_2 = v_1 - \frac{h_0 g}{a} = v_1 - \Delta v. \tag{15a}$$

The velocity–time curve for Periods 2 and 3 (Fig. 3), has been drawn according to the assumption of Equation (22) for a cycle having three pressure surges.

The amount of water pumped per cycle is $q_s = \int_0^{t_3} wAvdt$, which may be expressed as the product of the specific weight of water, the cross-sectional area of the drive pipe and the area under the velocity–time curve of Fig. 3. The values of the several parts of the area of $\int_0^{t_3} vdt$ are indicated on Fig. 3.

The evaluation of areas under the horizontal portions of the velocity–time curve is obvious. To simplify the resulting expressions, the quantity t' is defined as

$$t' = \frac{2L_1}{a} - t_2. \tag{23}$$

The area under each of the sloping portions of the curve is evaluated in the following manner:

The velocity–time curve for period 2 is shown in Fig. 4. During Period 2, from Equation (14)

$$\frac{dh'}{Y} = Avdt.$$

Then $\int \frac{dh'}{YA} = \int vdt$ = the area under the velocity–time curve for any part of Period 2. Integration of this equation gives

$$\int_H^{h'} \frac{dh'}{YA} = \frac{h' - H}{YA}.$$

Then the shaded area B in Fig. 4 is

$$B = v_1 t - \frac{h' - H}{YA}.$$

But from Equation (12)

$$(h' - H) = \frac{a}{g}(v_1 - v).$$

Then

$$B = v_1 t - \frac{a}{AgY}(v_1 - v) = v_1 t - Z(v_1 - v).$$

From Equation (19),

$$v = v_1 e^{-t/Z}. \tag{24}$$

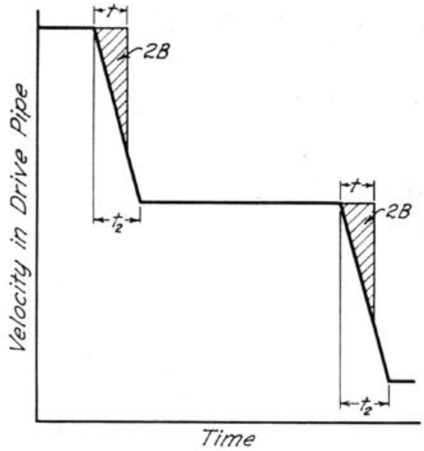

FIG. 5. RELATION OF VELOCITY IN DRIVE PIPE AT RAM TO TIME DURING A PART OF PERIOD 3

Hence

$$B = v_1 t - v_1 Z + v_1 Z e^{-t/Z} = v_1 [t - Z(1 - e^{-t/Z})]. \qquad (25)$$

During Period 2, the decrease in velocity Δv is due to the generation of a pressure wave (Equation (11)); during the sloping portions of the velocity–time curve of Period 3, (see Fig. 5) a pressure wave of identical form is being dissipated and another of equal magnitude is being generated in an equal time interval t_2, reducing the velocity by $2\Delta v$. Therefore the velocity changes which occur at corresponding times during these portions of Period 3 must be twice as great as those during Period 2. Accordingly, the shaded areas of Fig. 5, which is a diagram of the velocity–time curve for part of Period 3, are each twice as great as the shaded area in Fig. 4 for corresponding times. That is, each shaded area of Fig. 5 is equal to

$$2B = 2v_1 [t - Z(1 - e^{-t/Z})]. \qquad (26)$$

If the shaded area of Fig. 5 be made to include the full width t_2 of a sloping portion of the curve,

$$2B = 2v_1 [t_2 - Z(1 - e^{-t_2/Z})]$$

$$2B = 2v_1 t_2 - 2Z(v_1 - v_1 e^{-t_2/Z}). \qquad (27)$$

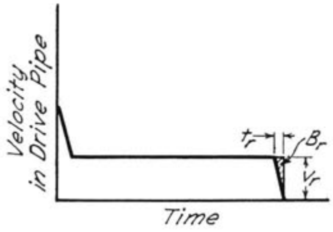

FIG. 6. RELATION OF VELOCITY IN DRIVE PIPE AT RAM TO TIME DURING FINAL PORTION OF PERIOD 3

With the aid of Equation (20), Equation (27) is seen to be equivalent to

$$2B = 2v_1t_2 - 2Z\,(v_1 - v_2)$$

or

$$2B = 2v_1t_2 - 2Z\Delta v. \tag{28}$$

Then for the part of surge 2 for which the velocity–time curve is sloping,

$$\begin{aligned}\int vdt &= (v_1 - \Delta v)t_2 - 2B \\ &= (v_1 - \Delta v)t_2 - (2v_1t_2 - 2Z\Delta v)\end{aligned}$$

$$\int vdt = 2Z\Delta v - t_2(v_1 + \Delta v). \tag{29}$$

Similarly, for surge 3, the corresponding area under the sloping part of the velocity–time curve is

$$\begin{aligned}\int vdt &= (v_1 - 3\Delta v)t_2 - 2B \\ &= (v_1 - 3\Delta v)t_2 - (2v_1t_2 - 2Z\Delta v)\end{aligned}$$

$$\int vdt = 2Z\Delta v - t_2(v_1 + 3\Delta v) \tag{30}$$

and so on.

For the last partial surge, the shaded area B_r in Fig. 6, according to Equation (26) is

$$B_r = 2v_1\left[t_r - Z\,(1 - e^{-t_r/Z})\right]$$

$$B_r = 2v_1t_r - 2Z\,(v_1 - v_1e^{-t_r/Z}). \tag{31}$$

The term in parentheses in Equation (31) is seen from Equation (24) to be the reduction in velocity during a time t_r of Period 2. Since the velocity decreases twice as fast during t_r as during a corresponding time of Period 2,

$$v_1 - v_1 e^{-t_r/Z} = \frac{v_r}{2}. \tag{32}$$

Substitution in Equation (31) gives

$$B_r = 2v_1 t_r - \frac{2Zv_r}{2}$$

and

$$\begin{aligned} \int_0^{t_r} v dt &= v_r t_r - B_r \\ &= v_r t_r - (2v_1 t_r - Zv_r) \end{aligned}$$

$$\int_0^{t_r} v dt = Zv_r - t_r\,(2v_1 - v_r). \tag{33}$$

The diagram (Fig. 3) has been drawn for only three surges; if the general consideration of N surges be made, the following equation results:

$$\begin{aligned} q_s = wA \int_0^{t_3} v dt = wA\,\Big\{ & \underbrace{[(v_1 - \Delta v)t']}_{\text{surge 1}} \\ & + \underbrace{[2Z\Delta v - (v_1 + \Delta v)t_2] + [(v_1 - 3\Delta v)t']}_{\text{surge 2}} \\ & + \underbrace{[2Z\Delta v - (v_1 + 3\Delta v)t_2] + [(v_1 - 5\Delta v)t']}_{\text{surge 3}} \\ & + \dots \underbrace{+\, Zv_r - (2v_1 - v_r)t_r}_{\text{final partial surge}} \Big\} \\ = wA\,[& Nv_1 t' + (N-1)2Z\Delta v + Zv_r \\ & - N^2 \Delta v t' - (N-1)^2 \Delta v t_2 - (N-1) v_1 t_2 \\ & - (2v_1 - v_r)t_r]. \end{aligned} \tag{34}$$

In Equation (34) Z may be determined from constants of the apparatus, (see Equations (18) and (13)) and v_1 and t_2 are found from Equations (8) and (20). The quantities Δv, N, v_r, and t_r remain to be determined.

From Equation (22)

$$\Delta v = \left[(h - H) + \frac{m(v_1 - \Delta v)}{4g} \right] \frac{g}{a}$$

whence

$$\Delta v = \frac{4g(h - H) + mv_1}{4a + m}. \tag{35}$$

An inspection of Fig. 3 shows that $2\Delta v > v_r \geq 0$, for if v_r were greater than $2\Delta v$, another surge would result, and v_r is prevented from becoming negative by the check valve. But from inspection of Fig. 3, it is seen that

$$v_r = v_1 - (2N - 1)\Delta v. \tag{36}$$

Then $$2\Delta v > v_1 - 2N\Delta v + \Delta v \geq 0.$$

Addition of $-v_1 - \Delta v$, to the above inequality gives

$$\Delta v - v_1 > -2N\Delta v \geq -v_1 - \Delta v.$$

Division of this result by $-2\Delta v$ gives

$$\frac{v_1 - \Delta v}{2\Delta v} < N \leq \frac{v_1 + \Delta v}{2\Delta v}. \tag{37}$$

It is evident that the difference between the first and last terms of inequality (37) is 1. Since N is necessarily an integer, it is uniquely determined.

With Δv and N determined from Equations (35) and (37), Equation (36) may be used to evaluate v_r.

Since the rate of change of velocity during the time t_r is the same as that during the other sloping portions of Period 3, and is twice as great as during Period 2, at corresponding times,

$$\left. v_r - v \right]_{\text{Period 3}} = \left. 2(v_1 - v) \right]_{\text{Period 2}}. \tag{38}$$

Equation (24) states that

$$v\Big]_{\text{Period 2}} = v_1 e^{-t/Z}.$$

Division of both sides by -1, and addition of v_1 to both sides gives

$$v_1 - v\Big]_{\text{Period 2}} = v_1(1 - e^{-t/Z}). \tag{39}$$

Substitution of Equation (39) in Equation (38) gives

$$v_r - v\Big]_{\text{Period 3}} = 2v_1(1 - e^{-t/Z}).$$

Hence

$$e^{t/Z} = \frac{2v_1}{2v_1 - v_r + v}. \tag{40}$$

But when $t = t_r$, $v = 0$. Therefore

$$e^{t_r/Z} = \frac{2v_1}{2v_1 - v_r}$$

or

$$t_r = Z \log_e \frac{2v_1}{2v_1 - v_r}. \tag{41}$$

Since each term in Equation (34) has now been evaluated, q_s may be computed.

By reference to Fig. 3, t_3 is determined as

$$t_3 = \frac{2L_1}{a} N - t_2 + t_r. \tag{42}$$

If the waste valve is considered inelastic, or $E_v = \infty$, then $Z = 0$, $t_2 = 0$, and Equation (34) reduces to the form given by O'Brien[2] and Gosline,

$$q_s = \frac{2L_1 A w}{a} (Nv_1 - N^2 \Delta v). \tag{43}$$

12. *Analysis of Period 4.*—At the end of Period 3, the velocity of the water at the check valve is of course zero, and the velocity of the water on the supply side of the pressure wave is $(v_1 - 2N\Delta v)$ which velocity may have any value from $+\Delta v$ (toward the ram) to $-\Delta v$ (away from the ram). The elastic energy stored in that part of the water column between the end of the pressure wave and the check valve is just enough to bring the whole column of water to a final velocity of $(v_1 - 2N\Delta v)$. By definition,

$$(v_1 - 2N\Delta v) = v_3. \tag{44}$$

There is also some energy stored in the waste valve disc due to its compression. Disregarding energy lost in internal hysteresis and pipe friction, the strain energy in the waste valve disc is expended in giving the water column a velocity toward the supply. The exact pressure and displacement variation is obscure; it may be noted, however, that if v_3 is positive, that is, toward the ram, the velocity must be reversed in direction before the waste valve can open. The reversal is similar to the reversal of velocity caused when a valve is suddenly closed in a straight length of pipe, so that the velocity produced away from the ram is the same in magnitude, but opposite in sense. If v_3 is negative, no such reversal is required.

The energy stored in water velocity and in compression of the waste valve disc is evaluated as follows:

$$\text{Energy stored in waste valve disc} = \tfrac{1}{2}(\text{load}) \times (\text{deflection}). \tag{45}$$

However,

$$\text{load} = wh_0A_v$$

and

$$\text{deflection} = \frac{\text{load}}{E_v} = \frac{wh_0A_v}{E_v}$$

Then

$$\text{Energy stored in waste valve disc} = \tfrac{1}{2}\, wh_0A_v \frac{wh_0A_v}{E_v}. \tag{46}$$

From Equation (22)

$$h_0 = \frac{\Delta va}{g}$$

$$\text{Energy stored in waste valve disc} = \tfrac{1}{2} \frac{w^2A_v{}^2a^2\Delta v^2}{g^2E_v}. \tag{47}$$

But since

$$Z = \frac{a}{AgY} = \frac{awA_v^2}{AgE_v}$$

$$\text{Energy stored in waste valve disc} = \frac{wA}{2g}\,aZ(\Delta v)^2 \qquad (48)$$

$$\text{Energy due to velocity of water column} = \tfrac{1}{2}(\text{mass}) \times (\text{velocity})^2 = \tfrac{1}{2}\frac{wAL}{g}v^2. \qquad (49)$$

The kinetic energy due to the velocity v_3, added to the strain energy stored in the waste valve disc, is assumed equal to the kinetic energy required to produce the velocity v_4. That is,

$$\frac{wA}{2g}\,Lv_3^2 + \frac{wA}{2g}\,aZ(\Delta v)^2 = \frac{wA}{2g}\,Lv_4^2$$

whence

$$v_4 = -\sqrt{v_3^2 + \frac{aZ}{L}(\Delta v)^2}\,. \qquad (50)$$

The time during which these energy transfers take place is not determined by the energy analysis, since the actual velocity variations are obscure. But it seems reasonable to assume that if v_3 is positive, so that a reversal takes place, this reversal will take a time $2L_1/a$. Therefore if v_3 is positive, t_4 will be $2L_1/a$ seconds longer than if v_3 is negative.

If v_3 is negative, it is evident that enough water must flow out of the valve box during Period 4 to make room for the expansion of the waste valve to its uncompressed size. Therefore,

flow out of the valve box = volume of expansion of waste valve disc. (51)

The volume of expansion of the waste valve disc must equal the volume of compression during Period 2. But in Period 2, since the volume of water flowing into the valve box must equal the volume of compression,

$$\text{volume of compression} = A\int_0^{t_2} v\,dt$$

for Period 2.

From Fig. 4,

$$\int_0^{t_2} v\,dt = v_1 t_2 - B$$

for Period 2. Evaluation of B from Equation (25) when $t = t_2$ gives

$$A\int_0^{t_2} v\,dt = A\left[v_1 t_2 - v_1\left\{t_2 - Z(1 - e^{-t_2/Z})\right\}\right] = AZ(v_1 - v_1 e^{-t_2/Z})$$

or

$$\text{volume of compression} = AZ(v_1 - v_1 e^{-t_2/Z}).$$

By Equation (24)

$$(v_1 - v_1 e^{-t_2/Z}) = v_1 - v_2 = \Delta v.$$

Hence

$$\text{volume of compression} = AZ\Delta v.$$

The total volume which flows out of the valve box during Period 4 must equal the product of the average velocity, the area of the pipe, and the time of the period. An approximate average velocity is $\dfrac{v_3 + v_4}{2}$.

Therefore,

$$\text{flow out of the valve box} = \frac{v_3 + v_4}{2} A t_4.$$

Substitution of these values in Equation (51) gives

$$-\frac{v_3 + v_4}{2} A t_4 = AZ\Delta v.$$

The negative sign on the left-hand side is necessary because velocity has been defined as negative when directed away from the ram. Thus, if v_3 is negative,

$$t_4 = -\frac{2Z\Delta v}{(v_3 + v_4)}. \tag{52a}$$

If v_3 is positive,

$$t_4 = -\frac{2Z\Delta v}{(v_4 - v_3)} + \frac{2L_1}{a}. \tag{52b}$$

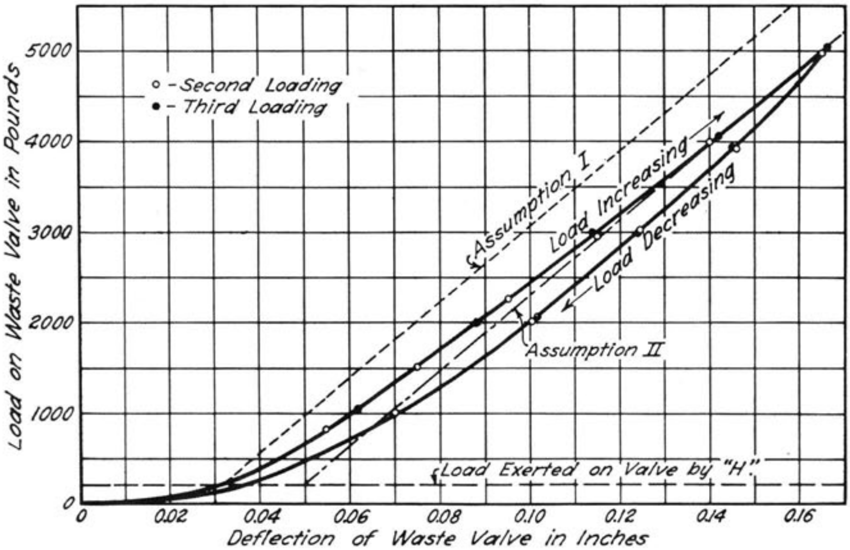

FIG. 7. LOAD–DEFLECTION TEST OF WASTE VALVE OF THE 4-INCH RAM

By Equations (50) and (52a) or (52b) the final velocity v_4 (negative) and the length (t_4) of Period 4 may be determined.

The preceding analysis has all been made on the assumption that the load-deflection curve for the waste valve is the straight line of assumption I of Fig. 7. Another assumption which might have been made is also shown in Fig. 7. Assumption II is probably closer to the actual curve, but introduces some inconvenience in analysis. Enough study of assumption II has been made to show the following differences in the two assumptions.

During Period 2, both assumptions give the same results, since the equations in Period 2 require only that the curve be a straight line with slope E_v, and both assumptions fulfill this requirement. Thus the value of t_2 is the same for both assumptions, and therefore the results in Period 3 are unaffected by the choice of assumption.

During Period 4, the energy analysis which gives the relation from which v_4 is calculated (Equation (50)) is the same for both assumptions, since the energy for any load on the valve is equal to the area under the load-deflection curve, and that area is the same for both assumptions.

The displacement of the water which must take place in order that the waste valve can open at the end of Period 4 is proportional to the actual displacement of the valve. The two assumptions give considerably different results for this quantity, as shown in Fig. 7. However, the time required to accomplish the displacement which assumption II would involve is never more than 0.005 seconds greater than the time (t_4) which assumption I requires for the 2-inch ram. For the 4-inch ram the difference is greater; for the worst case of very low pumping head and very low v_0, the difference in the duration of Period 4 as determined according to the two assumptions is 0.042 seconds, or 6 per cent of the total time t_s. For the condition of maximum efficiency for any v_0, the difference is never greater than 3 per cent of the total time t_s. The difference decreases rapidly with the increase of either pumping head or v_0.

Of course, during Periods 1, 5, and 6 the assumption used makes no difference, since no deflection of the waste valve occurs.

The convenience of assumption I seems to justify its use, since the errors produced by using it have been shown to be comparatively small.

13. *Analysis of Period 5.*—During this period the water is considered incompressible. Pressure fluctuations during Periods 5 and 6 are small enough to make the effects of compressibility negligible. The velocity in the drive pipe is therefore uniform all along the pipe, and is at first negative, equal to v_4. The water is continually accelerated toward the ram by the unbalanced supply pressure, so that the velocity decreases to zero, then increases again toward the ram, until the valve box is refilled and water begins to waste.

Pipe friction is disregarded during this period. An analysis was made taking friction into account, and numerical calculations showed that for the test ranges of the rams used experimentally, the values of v_5 and t_5, as determined by disregarding friction, were not more than 5 per cent greater than the values determined by taking friction into account.

Consideration of the forces on the water column during Period 5 gives

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$wAH = \frac{wAl}{g} \times \frac{dv}{dt}$$

where l is the variable length of the water column in the drive pipe.

However, the variation in l is quite small compared to the maximum length L. The substitution $l = L$ gives

$$dt = \frac{L}{gH}\,dv.$$

$$\int_0^{t_5} dt = \int_{v_4}^{v_5} \frac{L}{gH}\,dv.$$

Therefore

$$t_5 = \frac{L}{gH}\,(v_5 - v_4).$$

Since, if friction is disregarded,

$$v_5 = -v_4 \tag{53}$$

$$t_5 = \frac{-2Lv_4}{gH} = \frac{2Lv_5}{gH}. \tag{54}$$

From Equations (53) and (54) respectively, v_5 and t_5 may be determined.

14. *Analysis of Period 6.*—Friction must be taken into account during this period, since there is appreciable loss both in drive pipe and waste valve, with higher velocities.

Consideration of the forces acting on the column of water in the drive pipe gives

Force = mass × acceleration

$$wA\left(H - \frac{v^2}{2g}\right) - wA\,(b + c)\,\frac{v^2}{2g} = wA\,\frac{L}{g}\,\frac{dv}{dt}.$$

The substitution $j = b + c + 1$ gives

$$dt = \frac{\dfrac{2L}{j}}{\dfrac{2gH}{j} - v^2}\,dv \tag{55}$$

$$\int_0^{t_6} dt = \int_{v_5}^{v_6} \frac{\frac{2L}{j}\,dv}{\frac{2gH}{j} - v^2}.$$

Therefore

$$t_6 = \frac{L}{j\sqrt{\frac{2gH}{j}}} \log_e \frac{\dfrac{\sqrt{\frac{2gH}{j}} + v_6}{\sqrt{\frac{2gH}{j}} - v_6}}{\dfrac{\sqrt{\frac{2gH}{j}} + v_5}{\sqrt{\frac{2gH}{j}} - v_5}}.$$

Substitution of v_0 for v_6 gives

$$t_6 = \frac{L}{j\sqrt{\frac{2gH}{j}}} \log_e \frac{\left(\sqrt{\frac{2gH}{j}} + v_0\right)}{\left(\sqrt{\frac{2gH}{j}} - v_0\right)} \frac{\left(\sqrt{\frac{2gH}{j}} - v_5\right)}{\left(\sqrt{\frac{2gH}{j}} + v_5\right)}. \tag{56}$$

From Equation (56) t_6 may be determined.

From Equation (55) it is seen that

$$\frac{dv}{dt} = \frac{\frac{2gH}{j} - v^2}{\frac{2L}{j}} \text{ at any time during Period 6.}$$

Since α_6 is defined as the acceleration of the water column at the end of Period 6 when the velocity is v_6, or its equivalent v_0,

$$\alpha_6 = \frac{\frac{2gH}{j} - {v_0}^2}{\frac{2L}{j}}. \tag{57}$$

TABLE 1
OUTLINE OF METHOD OF COMPUTATION
Beginning at Period 1

Period	Velocity-Functions	Equation No.	Time-Functions	Equation No.	Displacement-Functions	Equation No.	Constants to be Determined
1	$v_1 = f(p, t_1)$	8	$t_1 = f(p)$	4	$Q_1 = f(p, t_1)$	10	S_0, J, w b, c, v_0, g, H, L, A
2	$v = f(p, v_1)$	15	$t_2 = f(p, v_1, v_2)$	20			L_1, h, m, A_v, E_v, a
3	$v_3 = f(p, v_1, v_2)$	44	$t_3 = f(p, v_1, v_2, t_2)$	42	$q_s = f(p, v_1, v_2, t_2)$	34	
4	$v_4 = f(p, v_1, v_2, v_3)$	50	$t_4 = f(p, v_1, v_2, v_3, v_4)$	52			
5	$v_5 = -v_4$	53	$t_5 = f(p, v_4)$	54			
6	$v_6 = v_0$		$t_6 = f(p, v_5)$	56	$Q_6 = f(p, v_5)$	58	

Also from the equation for dv/dt, the quantity wasted during Period 6 is

$$Q_6 = Aw\int vdt = Aw\int_{v_5}^{v_0} \frac{\dfrac{2L}{j}vdv}{\dfrac{2gH}{j} - v^2}.$$

Therefore

$$Q_6 = \frac{wAL}{j}\log_e \frac{\dfrac{2gH}{j} - v_5^2}{\dfrac{2gH}{j} - v_0^2}. \tag{58}$$

15. *Summary of Analysis.*—Inspection of all the equations so far developed will show that though they have been worked out assuming a horizontal drive pipe, they are independent of the slope of the drive pipe as long as all parts of the drive pipe are under a sufficient absolute pressure to prevent cavitation.

To show the continuity of the analysis, and the sequence in which computations must be made, Table 1 is presented. The symbol p is understood to be indicative of constants of the setting of the ram which can be measured or assumed.

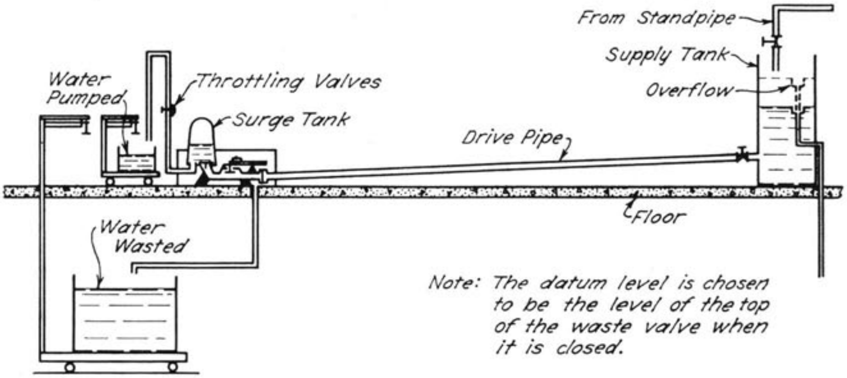

FIG. 8. DIAGRAM OF LABORATORY SETTING

Now that the analysis has been completed in each period, the results are combined to obtain the desired quantities Q and q.

$$t_s = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 \tag{59}$$

$$Q_s = Q_1 + Q_6 \tag{60}$$

$$Q = \frac{60Q_s}{t_s} \tag{61}$$

$$q = \frac{60q_s}{t_s}. \tag{62}$$

III. EXPERIMENTAL STUDY

16. *Pumping Tests.*—Two rams were used to obtain experimental performance data. Both were manufactured by the Rife Hydraulic Manufacturing Company; one which was designated by them as size No. 40A was fitted with a 4-in. drive pipe 55.5 ft. long; the other, which was size No. 20B, was fitted with a 2-in. drive pipe 54.8 ft. long. Figure 8 is a diagram of the laboratory setting which was similar for the two rams. Figures 9 and 10 are views of the 4-in. ram as set up for operation.

Water was supplied from a constant-head tank, equipped with two overflow pipes at different levels so that two supply pressures were available. Only the higher of these, approximately nine feet,

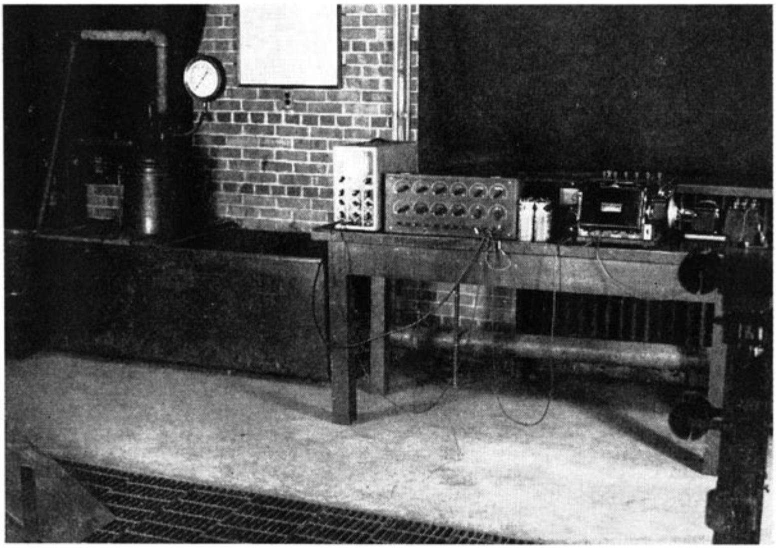

FIG. 9. VIEW OF RAM AND OSCILLOGRAPH AS SET UP IN THE LABORATORY

was used in the performance tests, since an unsteady operation was observed when operation at the five-foot supply head was attempted. The supply pressure was measured by a manometer fitted with a gage reading to 0.01 ft. Though some surges occurred in the manometer due to the pounding of the ram and the shaking of the supply tank, the average supply pressure did not vary more than 0.01 ft. during any test.

The delivery pressure was controlled by throttling valves in the delivery pipe. This pressure was measured by a Bourdon gage of suitable range, calibrated by a dead-weight tester before and after use. The readings of each gage were corrected for errors in indication and for the height of the gage above datum. Particularly at high delivery pressures, some difficulty was experienced in maintaining a constant discharge pressure, and some adjustment of the valves was found necessary during the progress of a test. The pressure was allowed to vary no more than one per cent on either side of the mean indicated pressure.

The water wasted during operation of the ram was caught in a box which surrounded the ram, and drained to a tank mounted on scales. The rate of wasting was determined by measuring, with an Eastman timer reading to half-seconds, the time required for wasting a given quantity of water. Both timer and scales were calibrated,

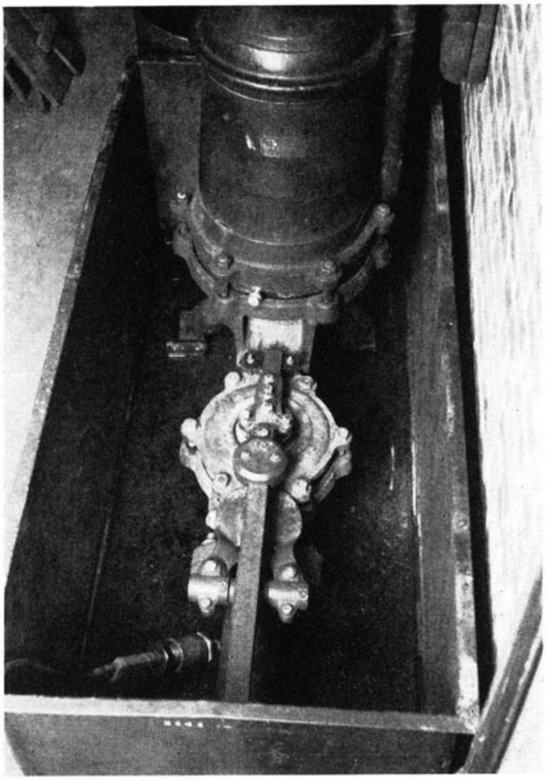

FIG. 10. VIEW OF RAM AND ELECTRIC PRESSURE GAGE AS SET UP IN THE LABORATORY

and no test of less than two minutes was made. The temperature of the water was approximately 70 deg. F.

The water pumped was measured in another tank, mounted on calibrated scales. The rate of pumping was determined by measuring with another calibrated timer the time required for pumping a given quantity of water. No test of less than 2½ minutes was made, except at extremely low delivery pressures, when the capacity of the tank limited the length of the test.

The frequency of the pumping cycle was determined by measuring, with one of the previously described timers, the time for 100 cycles. For all the tests the frequency was less than 100 cycles per minute.

The velocity, v_0, in the drive pipe necessary to begin waste valve closure, was controlled by two adjustments. The length of the

TABLE 2
PERFORMANCE DATA

Series 1

2-in. ram; $H = 9.2$ ft.; $v_0 = 4.48$ ft. per sec.; $S_0 = 0.0269$ ft.; weight to the left

Test Number	h ft.	Q lb. per min.	q lb. per min.	Rankine Efficiency* per cent	Q_s lb. per cycle	q_s lb. per cycle	t_s sec. per cycle
711	24.5	176.0	30.6	28.9	4.40	0.765	1.450
710	34.0	182.2	25.0	36.9	4.38	0.600	1.440
709	43.5	184.9	21.0	42.4	4.38	0.498	1.420
708	53.0	185.2	17.9	46.0	4.36	0.421	1.415
707	63.5	187.5	15.7	49.4	4.34	0.364	1.390
706	73.5	187.5	13.8	51.5	4.33	0.319	1.385
705	83.5	187.5	12.2	52.6	4.33	0.282	1.385
704	93.5	184.6	10.9	54.1	4.30	0.254	1.395
703	103.5	185.2	9.9	54.7	4.31	0.231	1.395
702	113.5	185.2	9.1	55.7	4.29	0.211	1.390
701	123.5	180.7	8.0	55.0	4.25	0.188	1.415
700	133.0	177.5	7.3	55.1	4.23	0.174	1.430
699	139	179.7	7.1	55.6	4.28	0.169	1.430
698	149	179.1	6.5	55.3	4.27	0.155	1.430
697	159	180.2	6.1	55.2	4.25	0.143	1.415
671	168	182.3	5.9	55.9	4.33	0.140	1.420
672	177	181.8	5.5	55.4	4.30	0.130	1.420
673	188	176.4	4.9	54.1	4.25	0.118	1.445
674	197	172.0	4.6	54.5	4.21	0.112	1.465
675	206	164.9	4.1	52.7	4.09	0.102	1.490
676	216	159.0	3.8	53.8	4.00	0.096	1.510
677	226	158.5	3.6	54.0	4.00	0.091	1.515
678	235	157.7	3.5	54.6	3.98	0.088	1.515
679	246	157.7	3.3	54.0	3.97	0.083	1.510
680	256	158.8	3.2	54.2	3.98	0.080	1.505
681	266	159.0	3.1	54.6	3.99	0.078	1.505
682	275	158.8	2.9	53.0	3.98	0.073	1.505
683	286	159.7	2.8	53.3	3.99	0.070	1.500
684	297	160.4	2.7	52.7	4.01	0.068	1.500
685	307	159.8	2.6	52.7	4.00	0.065	1.500
686	317	161.3	2.5	54.0	4.01	0.062	1.495
687	327	161.3	2.3	49.3	4.01	0.057	1.495
688	337	161.3	2.2	48.6	4.01	0.055	1.495
689	346	160.8	2.1	47.8	3.99	0.052	1.490
690	357	160.4	1.9	44.7	3.99	0.047	1.495
691	367	157.4	1.8	44.5	3.94	0.045	1.500
692	378	154.1	1.6	41.7	3.89	0.040	1.510
693	387	151.0	1.5	40.8	3.82	0.039	1.515
694	397	148.3	1.4	39.9	3.78	0.036	1.530
695	407	144.9	1.2	35.8	3.74	0.031	1.545
696	418	142.1	1.1	34.4	3.68	0.029	1.555

Series 2

2-in. ram; $H = 9.2$ ft.; $v_0 = 2.93$ ft. per sec.; $S_0 = 0.0269$ ft.; weight to the right

712	29.0	100.0	22.4	48.1	1.53	0.343	0.920
713	34.0	102.3	20.2	53.2	1.54	0.305	0.905
714	43.5	105.5	16.7	60.8	1.55	0.246	0.885
715	53.0	105.8	13.8	62.1	1.55	0.202	0.880
716	63.5	104.4	11.5	65.0	1.53	0.169	0.885
717	73.5	105.3	10.1	67.0	1.54	0.147	0.875

*Rankine efficiency $= \dfrac{q\,(h - H) \times 100}{QH}$.

TABLE 2 (CONTINUED)

PERFORMANCE DATA

Series 2 (Concluded)

2-in. ram; $H = 9.2$ ft.; $v_0 = 2.93$ ft. per sec.; $S_0 = 0.0269$ ft.; weight to the right

Test Number	h ft.	Q lb. per min.	q lb. per min.	Rankine Efficiency* per cent	Q_s lb. per cycle	q_s lb. per cycle	t_s sec. per cycle
718	83.5	102.1	8.5	67.2	1.52	0.127	0.895
719	93.5	100.5	7.5	68.3	1.50	0.112	0.900
720	103.5	100.5	6.8	69.4	1.50	0.102	0.900
721	113.5	101.8	6.2	69.2	1.53	0.093	0.900
722	123.5	99.9	5.5	68.5	1.50	0.082	0.900
723	133.0	94.8	4.7	66.7	1.45	0.072	0.920
724	139	92.9	4.4	66.6	1.44	0.068	0.930
725	150	87.7	3.7	65.0	1.39	0.059	0.950
726	160	87.6	3.5	66.0	1.39	0.056	0.955
727	169	87.0	3.3	66.3	1.39	0.053	0.955
728	178	86.5	3.1	65.8	1.37	0.049	0.950
729	188	85.6	2.8	65.0	1.35	0.045	0.950
730	197	86.5	2.6	63.0	1.36	0.042	0.945
731	207	87.0	2.5	61.2	1.37	0.039	0.945
732	216	87.6	2.3	59.2	1.37	0.036	0.940
733	226	87.4	2.1	56.8	1.37	0.033	0.940
734	235	86.8	1.9	53.8	1.37	0.030	0.950
735	246	85.2	1.7	51.2	1.36	0.027	0.955
736	256	82.8	1.5	48.5	1.33	0.024	0.960
737	266	80.0	1.3	45.3	1.30	0.021	0.975
738	276	78.5	1.2	44.6	1.30	0.020	0.990
739	285	75.6	1.0	39.8	1.28	0.017	1.015
740	296	73.8	0.9	36.8	1.27	0.015	1.035
741	307	69.3	0.7	34.8	1.21	0.013	1.045
742	317	63.8	0.5	26.9	1.12	0.009	1.055
743	327	58.8	0.3	19.8	1.05	0.006	1.075
744	337	53.6	0.2	11.0	0.97	0.003	1.085

Series 3

2-in. ram; $H = 9.2$ ft.; $v_0 = 1.77$ ft. per sec.; $S_0 = 0.0108$ ft.; weight to center.

745	24.5	54.6	16.1	48.7	0.62	0.182	0.680
746	34.0	57.8	12.3	56.8	0.63	0.133	0.650
747	43.5	58.8	9.6	61.2	0.61	0.100	0.625
748	53.0	58.3	7.4	60.3	0.60	0.076	0.615
749	63.5	58.5	6.2	62.9	0.60	0.064	0.610
750	73.5	55.2	5.2	66.2	0.57	0.054	0.615
751	83.5	54.4	4.2	63.8	0.57	0.045	0.625
752	93.5	53.5	3.4	57.9	0.57	0.036	0.635
753	103.5	50.4	2.9	58.8	0.54	0.031	0.640
754	113.5	49.0	2.6	59.8	0.53	0.028	0.655
755	123.5	48.2	2.3	60.9	0.53	0.026	0.665
756	133.0	48.1	2.1	58.5	0.53	0.023	0.665
757	139	48.5	2.0	58.5	0.53	0.022	0.660
758	150	48.3	1.7	54.7	0.53	0.019	0.660
759	160	46.8	1.4	50.4	0.52	0.016	0.670
760	170	44.9	1.1	45.5	0.50	0.013	0.670
761	179	44.0	0.9	40.6	0.50	0.011	0.685
762	189	41.3	0.7	32.5	0.48	0.008	0.700
763	198	38.0	0.4	22.9	0.45	0.005	0.705
764	207	34.9	0.2	10.3	0.42	0.002	0.725

*Rankine efficiency $= \dfrac{q\,(h - H) \times 100}{QH}$.

TABLE 2 (CONTINUED)
PERFORMANCE DATA

Series 4

4-in. ram; $H = 9.0$ ft.; $v_0 = 3.19$ ft. per sec.; $S_0 = 0.0301$ ft.; weight to the left

Test Number	h ft.	Q lb. per min.	q lb. per min.	Rankine Efficiency* per cent	Q_s lb. per cycle	q_s lb. per cycle	t_s sec. per cycle
213	23.5	409	146.2	57.6	8.53	3.05	1.250
214	33.5	438	106.5	66.1	8.40	2.04	1.150
215	43.0	449	81.3	68.5	8.28	1.50	1.105
216	52.5	455	64.8	69.0	8.35	1.19	1.100
217	63.0	452	52.0	69.1	8.22	0.946	1.090
218	73.0	453	44.5	69.8	8.27	0.812	1.095
219	83.0	445	37.9	70.0	8.13	0.692	1.095
220	93.0	433	32.0	69.0	7.95	0.588	1.100
221	103.0	422	27.2	67.4	7.95	0.513	1.130
222	113.0	412	24.4	68.3	7.81	0.462	1.135
223	122.5	404	21.6	67.4	7.68	0.411	1.140
224	132.5	395	19.1	66.2	7.57	0.366	1.150
225	139	388	17.8	66.5	7.51	0.345	1.160
226	150	374	15.1	63.2	7.34	0.296	1.175
227	160	356	12.5	58.9	7.12	0.250	1.200
228	170	348	11.3	57.8	7.03	0.227	1.210
229	179	335	9.7	55.1	6.78	0.198	1.215
230	189	328	8.9	54.1	6.67	0.181	1.220
231	198	322	8.1	52.9	6.55	0.165	1.220
232	207	317	7.5	51.8	6.53	0.154	1.235
233	217	311	6.8	50.0	6.46	0.140	1.245
234	227	304	6.0	47.1	6.31	0.123	1.245
235	236	297	5.2	44.1	6.16	0.108	1.245
236	247	286	4.4	40.2	5.99	0.091	1.255
237	258	279	3.9	38.5	5.90	0.082	1.270
238	267	271	3.4	36.1	5.80	0.073	1.285
239	276	270	2.9	32.1	5.83	0.063	1.295
240	287	256	2.1	25.5	5.57	0.046	1.305
241	298	243	1.5	19.8	5.33	0.033	1.315
242	309	234	1.0	14.8	5.19	0.023	1.330
243	318	225	0.6	9.6	5.00	0.014	1.335
244	328	219	0.2	3.6	4.91	0.005	1.345

Series 5

4-in. ram; $H = 9.0$ ft.; $v_0 = 2.25$ ft. per sec.; $S_0 = 0.0204$ ft.; weight to the left

258	24.0	267	113.3	71.0	4.20	1.786	0.945
259	33.5	291	79.0	73.8	4.17	1.131	0.860
260	43.0	296	58.8	75.1	4.12	0.818	0.835
261	53.0	297	45.5	74.8	4.06	0.621	0.820
262	62.5	290	36.4	74.6	3.98	0.499	0.825
263	72.5	282	29.5	73.9	3.92	0.410	0.835
264	82.5	267	23.5	71.7	3.77	0.331	0.845
265	93.0	262	20.0	71.5	3.77	0.289	0.865
266	103.0	256	17.0	69.4	3.73	0.248	0.875
267	113.0	250	14.2	66.6	3.75	0.213	0.900
268	123.0	240	11.6	61.0	3.68	0.177	0.920
269	132.5	231	9.8	58.3	3.60	0.153	0.935
270	138	230	9.1	56.8	3.60	0.143	0.940
271	149	220	7.8	54.8	3.50	0.123	0.945
272	159	216	6.8	52.8	3.45	0.109	0.960
273	168	210	5.9	50.1	3.39	0.096	0.970
274	178	203	5.1	48.8	3.32	0.083	0.980
275	188	195	4.0	41.1	3.20	0.066	0.985
276	197	187	3.3	36.5	3.09	0.054	0.990
277	206	177	2.5	30.7	2.93	0.041	0.995
278	216	172	1.8	23.8	2.90	0.030	1.010
279	226	165	1.1	14.6	2.78	0.018	1.010
280	236	157	0.7	11.4	2.66	0.012	1.015
281	246	151	0.2	3.0	2.59	0.003	1.030

*Rankine efficiency $= \dfrac{q\,(h - H) \times 100}{QH}$.

TABLE 2 (CONCLUDED)
PERFORMANCE DATA
Series 6
4-in. ram; $H = 9.0$ ft.; $v_0 = 1.35$ ft. per sec.; $S_0 = 0.0164$ ft.; weight to the right

Test Number	h ft.	Q lb. per min.	q lb. per min.	Rankine Efficiency* per cent	Q_s lb. per cycle	q_s lb. per cycle	t_s sec. per cycle
245	24.5	174	72.0	71.2	1.96	0.810	0.675
246	34.5	184	48.4	74.6	1.93	0.508	0.630
247	44.5	184	34.5	74.1	1.87	0.351	0.610
248	54.5	175	25.6	73.9	1.78	0.260	0.610
249	64.0	160	18.8	71.9	1.65	0.192	0.615
250	74.5	150	14.2	68.9	1.58	0.149	0.630
251	85.0	142	11.1	66.0	1.53	0.119	0.645
252	95.5	131	8.0	58.8	1.41	0.089	0.665
253	107.0	126	6.2	53.7	1.41	0.069	0.675
254	110	121	5.4	50.3	1.36	0.061	0.675
255	118	115	4.3	45.3	1.31	0.049	0.685
256	127	104	2.9	35.8	1.21	0.033	0.695
257	138	93	1.5	25.0	1.09	0.019	0.700

*Rankine efficiency $= \dfrac{q\,(h - H) \times 100}{QH}$.

stroke of the waste valve was adjustable by means of two screws (L in Fig. 1). The counterweight (M in Fig. 1) was adjustable along the rocker arm of the waste valve. Both of these adjustments affect v_0. For each ram, a series of tests was made with each of three different values of v_0. In each series the delivery pressure was varied in approximately equal increments over the whole range of the ram, all other factors being held constant.

The results of these series of tests are presented in Table 2, and are shown graphically in Figs. 11 to 14 inclusive.

17. *Determination of Constants.*—The following constants were required (see Table 1):

$$S_0, J, m, b, c, v_0, E_v, L, L_1, A, A_v, a.$$

S_0, the length of stroke of the waste valve, was determined by attaching a steel rod to the center of the waste valve disc, so that it extended up to an Ames dial graduated in thousandths of an inch, mounted on the box surrounding the ram. Since it was difficult to determine the exact point at which the waste valve first touched its seat, and since the valve did not strike its seat at all points at once, S_0 was arbitrarily defined as the distance between the open position of the waste valve and its position when held closed by a pressure head of approximately nine feet of water. This value was easily determined with the Ames dial.

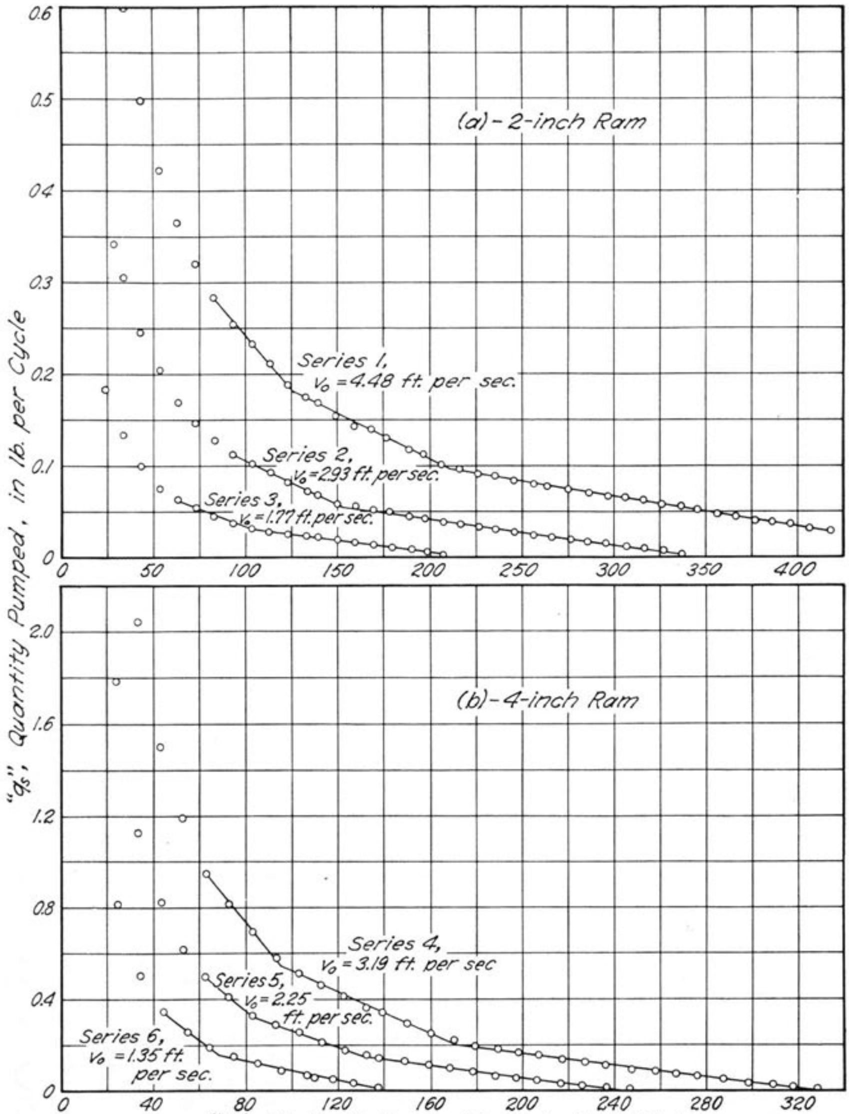

FIG. 11. RELATION OF q_s, THE QUANTITY PUMPED PER CYCLE, TO h, THE STATIC DELIVERY HEAD, AS DETERMINED BY EXPERIMENT FOR THE 2-INCH RAM AND THE 4-INCH RAM

A brass stylus was attached to the upper end of the steel rod used in the determination of S_0 in such a manner that it could be brought to bear on a strip of sensitized paper mounted on a drum

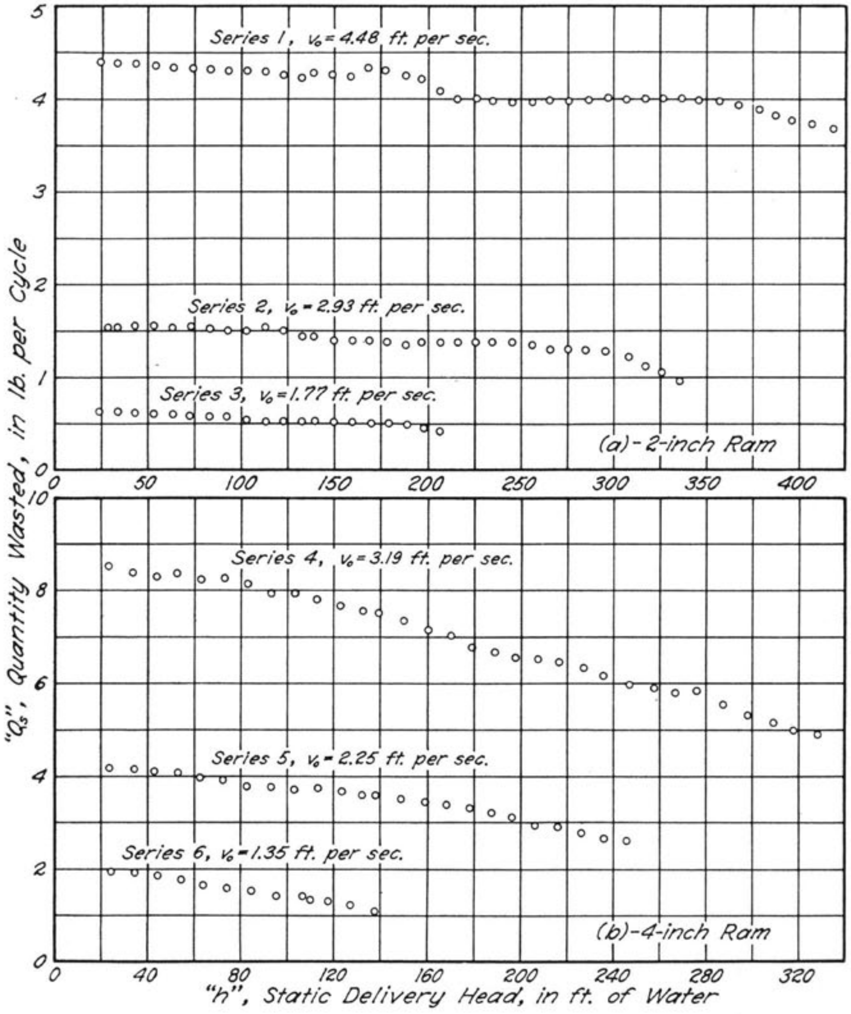

FIG. 12. RELATION OF Q_s, THE QUANTITY WASTED PER CYCLE, TO h, THE STATIC DELIVERY HEAD, AS DETERMINED BY EXPERIMENT FOR THE 2-INCH RAM AND THE 4-INCH RAM

which revolved at constant speed. Records of the relation of waste valve movement to time were thus obtained for a number of conditions of operation. Four of these records are reproduced in Fig. 15. The results of measurements of the length of time of waste valve closure from more than eighty of these records from the 4-in. ram are summarized in Table 3. These results apparently justified the assumption that t_1 could be represented by a function of $\sqrt{S_0}$

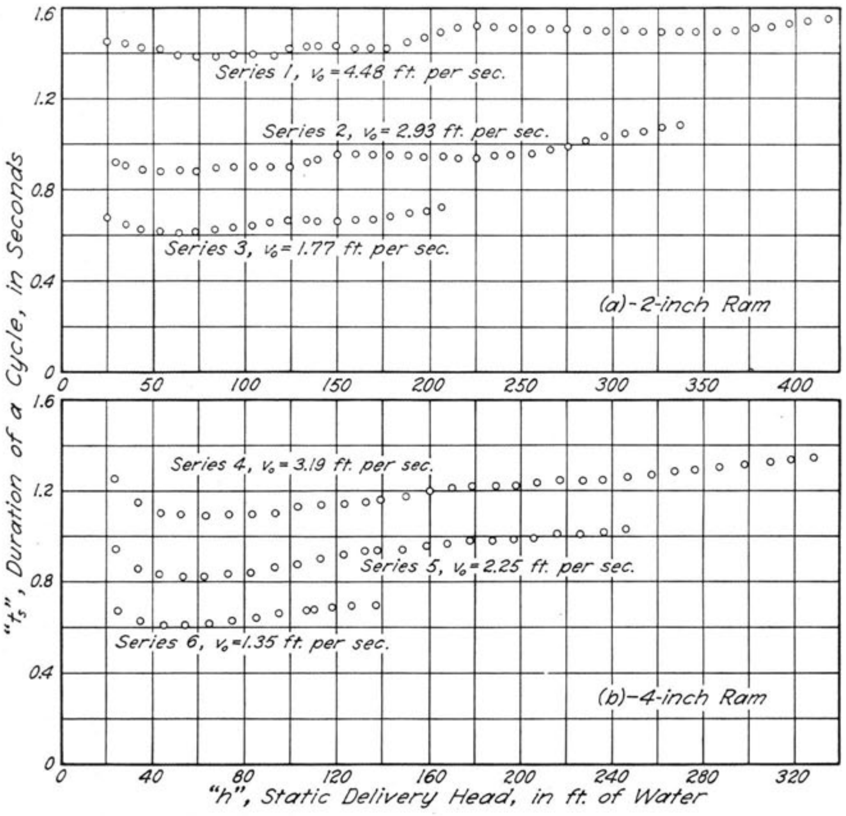

FIG. 13. RELATION OF t_s, THE DURATION OF A CYCLE, TO h, THE STATIC DELIVERY HEAD, AS DETERMINED BY EXPERIMENT FOR THE 2-INCH RAM AND THE 4-INCH RAM

alone, without reference to the counterweight position or supply or delivery pressure (Equation (4)). This assumption established the form of the function. The constant J was then evaluated for each ram by using the amounts actually wasted as a criterion of how long the closure period should be. In Table 4 are compiled a group of measurement results from the displacement–time diagrams for the waste valve, and the corresponding calculated values; it is evident that the discrepancies are considerable. The difficulty of precise determination of the point on the autographic records where the waste valve begins to move is probably the cause of part of these discrepancies.

The friction constants m, b, and c were determined by steady-flow tests, head loss being measured by a water manometer. To

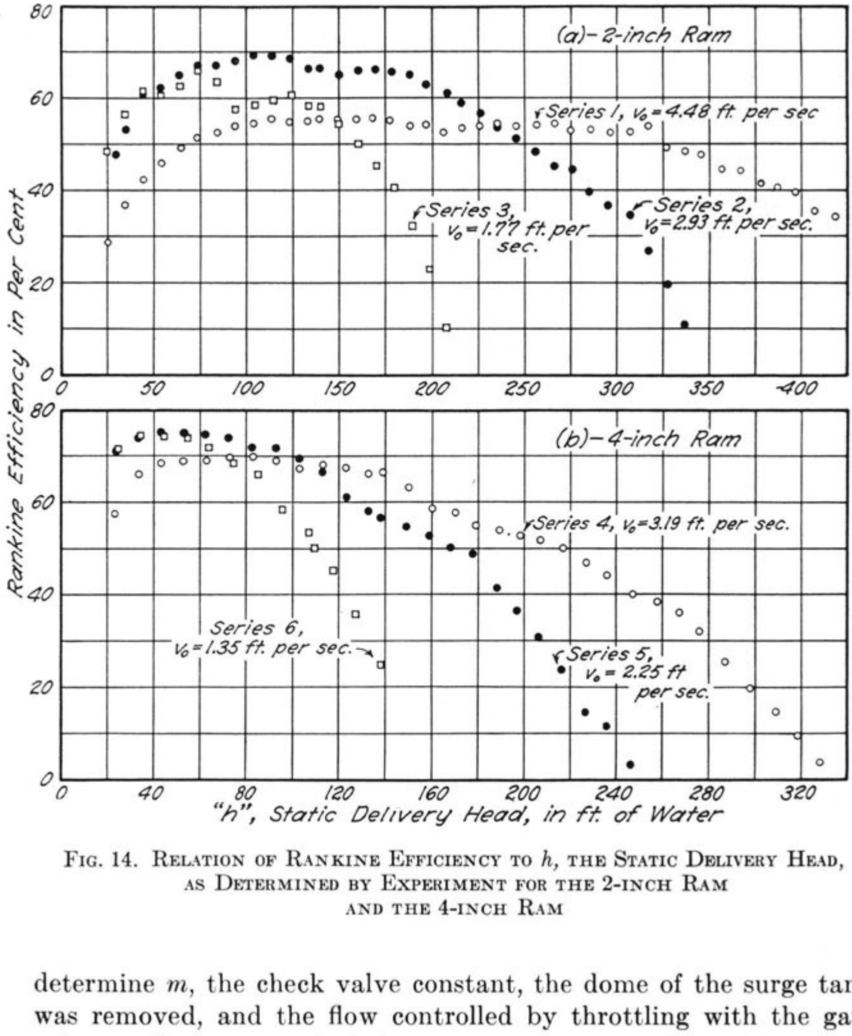

FIG. 14. RELATION OF RANKINE EFFICIENCY TO h, THE STATIC DELIVERY HEAD, AS DETERMINED BY EXPERIMENT FOR THE 2-INCH RAM AND THE 4-INCH RAM

determine m, the check valve constant, the dome of the surge tank was removed, and the flow controlled by throttling with the gate valve at the upper end of the drive pipe. The values of head loss were then plotted as ordinates and the corresponding values of velocity as abscissas in rectangular coordinates, and the slope of the straight line which best fitted the plotted points was multiplied by $2g$ to obtain m. To determine b, the friction coefficient of the drive pipe, the flow was controlled by blocking the waste valve partly open. To determine c, the friction coefficient of the waste valve, the flow was controlled by throttling at the upper end of the drive pipe, and the velocity always kept below that value v_0 which would cause the waste valve to close.

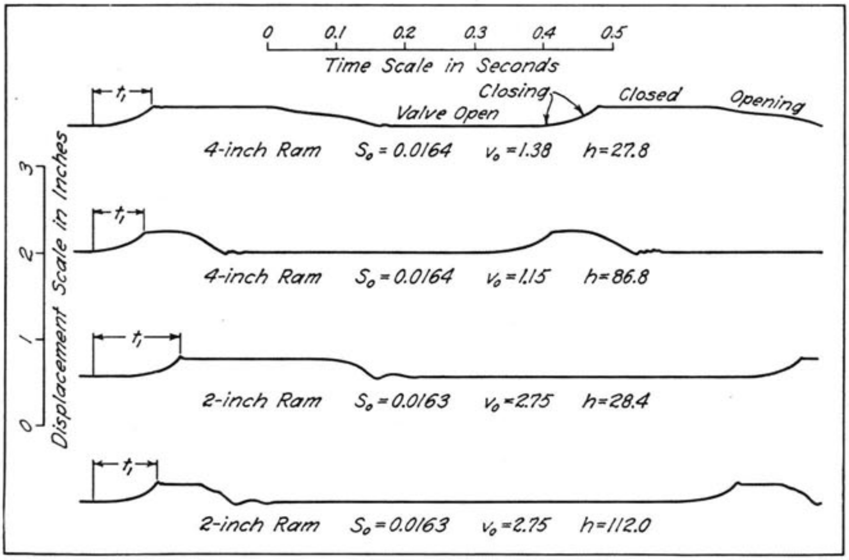

FIG. 15. DISPLACEMENT–TIME DIAGRAMS FOR THE WASTE VALVE

After c was determined by steady-flow test, the velocity v_0 was determined by slowly increasing the velocity, and noting by means of a water manometer the pressure head in the valve box just before closure of the valve. The following relation then determined v_0:

$$v_0 = \sqrt{\frac{2g}{c} \times \text{(head for valve closure).}}$$

TABLE 3
RESULTS OF MEASUREMENT OF t_1 FROM DISPLACEMENT–TIME DIAGRAMS FOR WASTE VALVE OF THE 4-INCH RAM

S_0	t_1 (measured)		
	Counterweight to the Left	Counterweight at the Center	Counterweight to the Right
0.0254	0.106	0.106	0.118
0.0228	0.094	0.104	0.095
0.0186	0.081	0.093	0.092
0.0164	0.081	0.077	0.084

TABLE 4

COMPARISON OF CALCULATED AND MEASURED VALUES OF t_1

S_0 ft.	Ram Size	t_1 Calculated from Equation 4 sec.	t_1 Measured sec.	Difference, per cent $\left(\frac{\text{calculated} - \text{measured}}{\text{measured}} \times 100\right)$
0.0269	2-in.	0.116	0.172	−32.6
0.0254	4-in.	0.130	0.109	+19.3
0.0228	4-in.	0.123	0.099	+24.2
0.0214	2-in.	0.104	0.127	−18.1
0.0186	4-in.	0.111	0.089	+24.8
0.0164	4-in.	0.104	0.080	+30.0
0.0163	2-in.	0.090	0.115	−21.7
0.0140	2-in.	0.084	0.088	− 4.5
0.0093	2-in.	0.068	0.068	0

The stiffness modulus of the waste valve, E_v, was determined by mounting the valve on its seat in a testing machine, applying a known load to the valve, and measuring its deflection with an Ames dial reading in thousandths of an inch. The values of total load on the valve were then plotted as ordinates and the corresponding values of deflection as abscissas, as in Fig. 7, and the slope of the straight line which best fitted the observed points was taken as E_v.

The physical dimensions L, L_1, A, and A_v were measured directly. The following formula, given by Gibson[4] for a pipe free to expand laterally and longitudinally was used to determine a:

$$a = \sqrt{\frac{g}{w\left[\frac{1}{K} + \frac{r}{2uE}\left(5 - \frac{4}{\sigma}\right)\right]}}$$

in which

g = acceleration of gravity, ft. per sec. per sec.
w = specific weight of water, lb. per cu. ft.
K = volume modulus of elasticity of water, lb. per sq. ft.
r = radius of pipe, ft.
u = thickness of pipe, ft.
E = modulus of elasticity of pipe, lb. per sq. ft.
σ = Poisson's ratio for pipe material.

All of these values are known quantities except r and u which were obtained by measurement.

18. *Pressure–Time Diagrams.*—A number of records of the relation of pressure to time were obtained by measuring the pressure

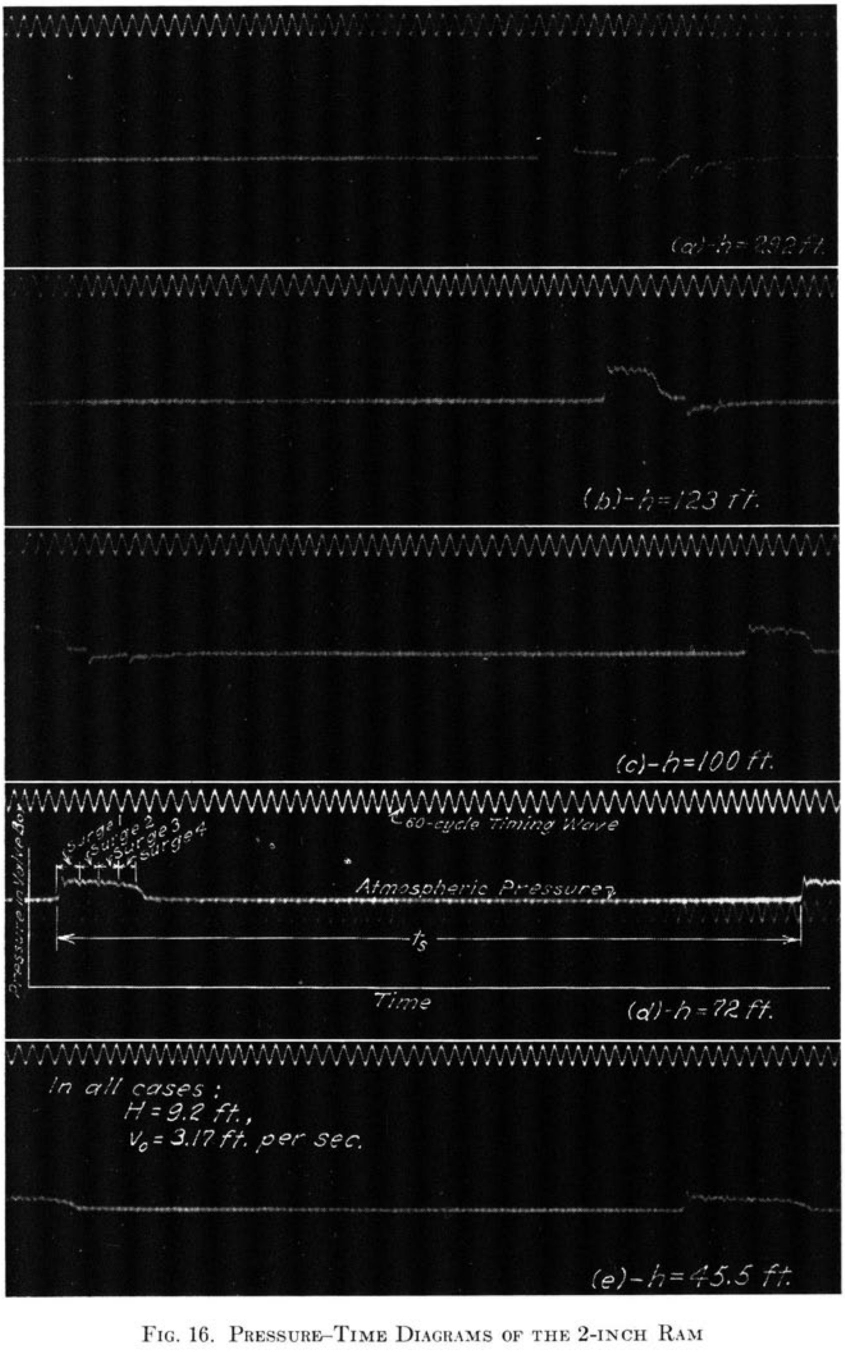

FIG. 16. PRESSURE–TIME DIAGRAMS OF THE 2-INCH RAM

in the drive pipe near the valve box (Fig. 10) with a Peters carbon-pile telemeter, and recording the reading with an oscillograph and traveling film. A few of these records are shown in Fig. 16, and in Fig. 16d the significant portions are labeled. The records are of interest because of their indication of the surges during Period 3, while the check valve was open. The average duration of a surge as indicated on these diagrams was 0.0245 sec.; the calculated value was 0.0251 sec. The quantitative value of the diagrams is questionable, because of the vacuum shown, up to 18 or 20 pounds per square inch, on some of the diagrams (see Fig. 16). It is probable that the shaking of the telemeter caused by the shock of the waste valve closure affected the reading.

19. *Discussion of Results.*—From a study of Fig. 11, it may be seen that an increase of the delivery head decreases the quantity pumped per cycle. Particularly for the 2-in. ram, and to a less noticeable degree for the 4-in. ram, it is evident that the relation between quantity pumped per cycle and delivery head appears to be made up of a series of straight lines rather than a smooth curve. It is also apparent that a decrease of v_0, the velocity required to begin waste valve closure, decreases the quantity pumped per cycle.

From Fig. 12 it may be seen that in general an increase of the delivery head decreases the quantity wasted per cycle. The decrease is greater for the 4-in. ram. For neither ram is the curve smooth; close examination with the aid of Fig. 17 will show that the values of delivery head at which the slope of the Q_s curve is most nearly horizontal correspond rather closely with the values at which the q_s curve changes slope. A decrease of v_0, the velocity required to begin waste valve closure, also decreases Q_s, the quantity wasted per cycle.

From Fig. 13 it may be seen that the duration of the cycle decreases with increasing delivery head, as long as the delivery head is below a certain value at which the duration of the cycle is a minimum; further increase in the delivery head in general increases the duration of the cycle, though not at a constant rate. The changes in duration of cycle are more marked for the 4-in. ram than for the 2-in. ram. From examination of Fig. 17 and the rest of the curves it appears that the values of delivery head at which the slope of the t_s curve becomes nearly horizontal correspond rather closely with the values at which the q_s curve changes slope. A decrease of v_0, the velocity required to begin waste valve closure, also decreases the duration of the cycle, or makes the ram beat faster.

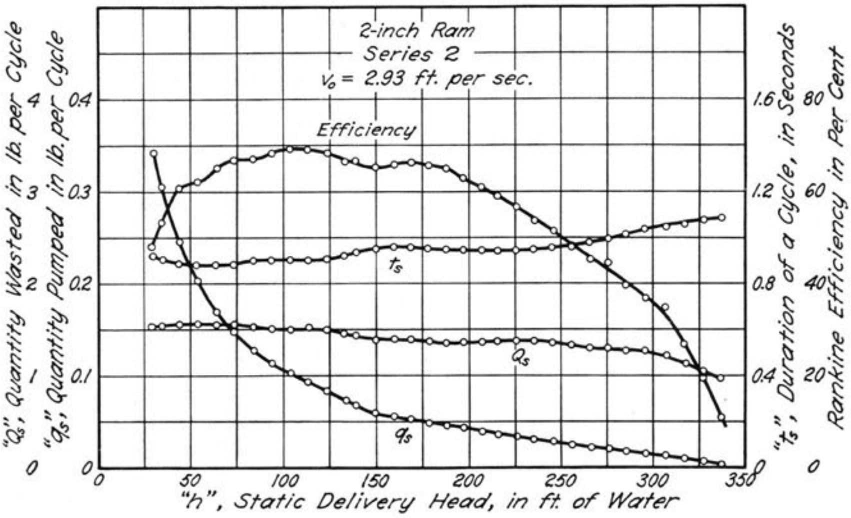

FIG. 17. RELATION OF q_s, THE QUANTITY PUMPED PER CYCLE, Q_s, THE QUANTITY WASTED PER CYCLE, t_s, THE DURATION OF A CYCLE, AND RANKINE EFFICIENCY, TO h, THE STATIC DELIVERY HEAD, SERIES 2, 2-INCH RAM

The variations of the Rankine efficiency, $\left[\frac{q(h-H)}{QH} \times 100\right]$, with delivery head are shown in Fig. 14. It is apparent that there are several maxima for each curve of the 2-in. ram, but that the efficiency curves for the 4-in. ram are smoother. Comparison will show (Fig.

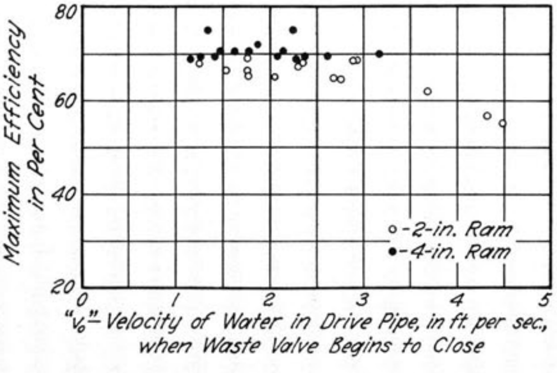

FIG. 18. RELATION OF MAXIMUM EFFICIENCY TO v_0, THE VELOCITY IN THE DRIVE PIPE REQUIRED TO BEGIN WASTE VALVE CLOSURE

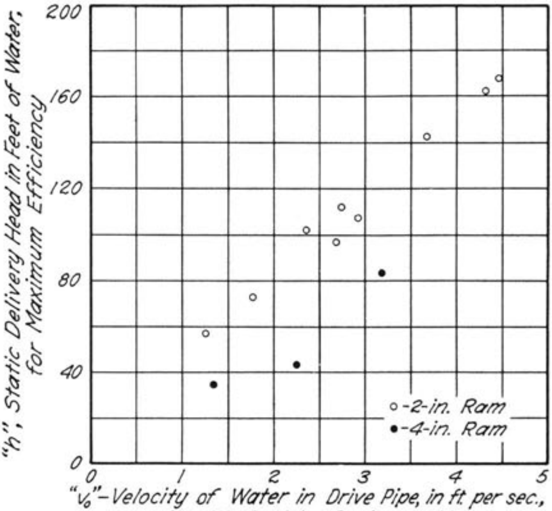

FIG. 19. RELATION OF STATIC DELIVERY HEAD h, AT WHICH MAXIMUM EFFICIENCY IS REACHED, TO v_0, THE VELOCITY IN THE DRIVE PIPE REQUIRED TO BEGIN WASTE VALVE CLOSURE

17) that the maxima of the efficiency curves occur between the values of delivery head at which the q_s curve changes slope.

The foregoing observations, particularly those referring to Fig. 17, are of interest when comparing the experimental results with the calculated performance curves (Section 21).

The variation of the maximum efficiency attained with different values of v_0, the velocity required to begin waste valve closure, is illustrated in Fig. 18. It appears that the maximum efficiency is not much affected by changes in v_0, except for extremely high values of v_0, at which the efficiency is somewhat lower.

The relation of the delivery pressure at which maximum efficiency occurs to the velocity v_0 is shown in Fig. 19. Though the points are somewhat scattered, it may be seen that the pressure at which the maximum efficiency occurs becomes higher as v_0 increases.

IV. Comparison of Theoretical and Experimental Results

20. *Examples of Computations.*—To illustrate the sequence of computations, and to show how the differences in the two rams affected the results, the following examples are shown.

2-in. Ram		4-in. Ram
	Constants	
3.10	v_0—ft./sec.	3.10
0.0161	S_0—ft.	0.0293
Left	Wt. Position	Left
4.0	J—ft./sec.^{2}	3.0
15.5	j—(dimensionless)	15.5
9.2	H—ft.	9.2
65	h—ft.	65
54.8	L—ft.	55.5
55.8	L_1—ft.	56.5
817	m—ft./sec.	96
3 870 000	E_v—lb./ft.	500 000
0.0233	A—ft.^{2}	0.0884
0.1043	A_v—ft.^{2}	0.371
4 450	a—ft./sec.	4 380
570 000	Y—1/ft.^{2}	58 200
0.00104	Z—sec.	0.0264

2-in. Ram		4-in. Ram
$t_1 = \sqrt{\dfrac{2 \times 0.0161}{4.0}} = 0.090$ sec.	Equation (4)	$t_1 = \sqrt{\dfrac{2 \times 0.0293}{3.0}} = 0.140$ sec.
$\alpha_6 = \dfrac{\dfrac{2 \times 32.2 \times 9.2}{15.5} - (3.10)^2}{\dfrac{2 \times 54.8}{15.5}} = 4.04$ ft./sec.^{2}	Equation (57)	$\alpha_6 = \dfrac{\dfrac{2 \times 32.2 \times 9.2}{15.5} - (3.10)^2}{\dfrac{2 \times 55.5}{15.5}} = 3.99$ ft./sec.^{2}
$v_1 = 3.10 + \dfrac{4.04 \times 0.090}{2} = 3.28$ ft./sec.	Equation (8)	$v_1 = 3.10 + \dfrac{3.99 \times 0.140}{2} = 3.38$ ft./sec.
$Q_1 = 62.4 \times 0.0233 \left[3.10 \times 0.090 + \dfrac{4.04 \times (0.09)^2}{3} \right] = 0.42$ lb./cycle	Equation (10)	$Q_1 = 62.4 \times 0.0884 \left[3.10 \times 0.14 + \dfrac{3.99 \times (0.14)^2}{3} \right] = 2.54$ lb./cycle
$\Delta v = \dfrac{4 \times 32.2(65-9.2)+817 \times 3.28}{4 \times 4450+817} = 0.530$ ft./sec.	Equation (35)	$\Delta v = \dfrac{4 \times 32.2(65-9.2)+96 \times 3.38}{4 \times 4380+96} = 0.430$ ft./sec.
$v_2 = 3.28 - 0.530 = 2.75$ ft./sec.	Equation (15a)	$v_2 = 3.38 - 0.430 = 2.95$ ft./sec.
$t_2 = 0.00104 \log_e \dfrac{3.28}{2.75} = 0.0002$ sec.	Equation (20)	$t_2 = 0.0264 \log_e \dfrac{3.38}{2.95} = 0.0036$ sec.
$N =$ first integer above $\dfrac{3.28 - 0.53}{2 \times 0.53}$; $N = 3$	Equation (37)	$N =$ first integer above $\dfrac{3.38 - 0.43}{2 \times 0.43}$; $N = 4$
$v_3 = 3.28 - 2 \times 3 \times 0.53 = +0.10$ ft./sec.	Equation (44)	$v_3 = 3.38 - 2 \times 4 \times 0.43 = -0.06$ ft./sec.

2-in. Ram		4-in. Ram
$v_r = 3.28 - (2 \times 3 - 1)\,0.53 = 0.63$ ft./sec.	Equation (36)	$v_r = 3.38 - (2 \times 4 - 1)\,0.43 = 0.37$ ft./sec.
$t_r = 0.00104 \log_e \dfrac{2 \times 3.28}{2 \times 3.28 - 0.63} = 0.0001$ sec.	Equation (41)	$t_r = 0.0264 \log_e \dfrac{2 \times 3.38}{2 \times 3.38 - 0.37} = 0.0015$ sec.
$t_3 = \dfrac{2 \times 55.8}{4450} \times 3 - 0.0002 + 0.0001 = 0.075$ sec.	Equation (42)	$t_3 = \dfrac{2 \times 56.5}{4380} \times 4 - 0.004 + 0.001 = 0.101$ sec.
$t' = \dfrac{2 \times 55.8}{4450} - 0.0002 = 0.0249$ sec.	Equation (23)	$t' = \dfrac{2 \times 56.5}{4380} - 0.0036 = 0.0222$ sec.
$q_s = (62.4 \times 0.0233) \left\{ \begin{bmatrix} 3 \times 3.28 \times 0.0249 \\ + (3-1)\,2 \times 0.00104 \\ \times 0.53 + 0.00104 \times 0.63 \end{bmatrix} \right.$	Equation (34)	$q_s = (62.4 \times 0.0884) \left\{ \begin{bmatrix} 4 \times 3.38 \times 0.0222 \\ + (4-1) \times 2 \times 0.0264 \\ \times 0.43 + 0.0264 \times 0.37 \end{bmatrix} \right.$
$\left. - \begin{bmatrix} (3)^2 \times 0.53 \times 0.0249 \\ + (3-1)^2 \times 0.53 \times 0.0002 \\ + (3-1) \times 3.28 \times 0.0002 \\ + (2 \times 3.28 - 0.63) \times 0.0001 \end{bmatrix} \right\} = 0.185$ lb./cycle		$\left. - \begin{bmatrix} 4^2 \times 0.43 \times 0.0222 \\ + (4-1)^2 \times 0.43 \times 0.0036 \\ + (4-1) \times 3.38 \times 0.0036 \\ + (2 \times 3.38 - 0.37) \times 0.0015 \end{bmatrix} \right\} = 0.914$ lb./cycle
$v_4 = -\sqrt{(0.10)^2 + \dfrac{4450 \times 0.00104}{54.8} (0.53)^2}$ $= -0.18$ ft./sec.	Equation (50)	$v_4 = -\sqrt{(-0.06)^2 + \dfrac{4380 \times 0.0264}{55.5} (0.43)^2}$ $= -0.62$ ft./sec.
$t_4 = -\dfrac{2 \times 0.00104 \times 0.53}{(-0.18 - 0.10)} = 0.004$ sec.	Equation (52)	$t_4 = -\dfrac{2 \times 0.0264 \times 0.43}{(-0.06 - 0.62)} + \dfrac{2 \times 56.5}{4380} = 0.059$ sec.
$v_5 = -(-0.18) = 0.18$ ft./sec.	Equation (53)	$v_5 = -(-0.62) = 0.62$ ft./sec.
$t_5 = \dfrac{2 \times 54.8 \times 0.18}{32.2 \times 9.2} = 0.067$ sec.	Equation (54)	$t_5 = \dfrac{2 \times 55.5 \times 0.62}{32.2 \times 9.2} = 0.232$ sec.

2-in. Ram		4-in. Ram
$t_6 = \dfrac{54.8}{15.5\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}}\log_e \left\{\dfrac{\dfrac{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}+3.10}{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}-3.10}}{\dfrac{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}+0.18}{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}-0.18}}\right\} = 0.595 \text{ sec.}$	Equation (56)	$t_6 = \dfrac{55.5}{15.5\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}}\log_e \left\{\dfrac{\dfrac{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}+3.10}{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}-3.10}}{\dfrac{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}+0.62}{\sqrt{\dfrac{2\times 32.2\times 9.2}{15.5}}-0.62}}\right\} = 0.522 \text{ sec.}$
$Q_6 = \dfrac{62.4\times 0.0233\times 54.8}{15.5}\log_e \left\{\dfrac{\dfrac{2\times 32.2\times 9.2}{15.5}-0.18^2}{\dfrac{2\times 32.2\times 9.2}{15.5}-3.10^2}\right\} = 1.48 \text{ lb./cycle}$	Equation (58)	$Q_6 = \dfrac{62.4\times 0.0884\times 55.5}{15.5}\log_e \left\{\dfrac{\dfrac{2\times 32.2\times 9.2}{15.5}-0.62^2}{\dfrac{2\times 32.2\times 9.2}{15.5}-3.10^2}\right\} = 5.50 \text{ lb./cycle}$
$t_s = 0.090 + 0 + 0.075 + 0.004 + 0.067 + 0.595 = 0.831 \text{ sec.}$	Equation (59)	$t_s = 0.140 + 0.004 + 0.101 + 0.059 + 0.232 + 0.522 = 1.058 \text{ sec.}$
$Q_s = 0.42 + 1.48 = 1.90 \text{ lb./cycle}$	Equation (60)	$Q_s = 2.54 + 5.50 = 8.04 \text{ lb./cycle}$
$Q = \dfrac{60\times 1.90}{0.831} = 137 \text{ lb./min.}$	Equation (61)	$Q = \dfrac{60\times 8.04}{1.058} = 456 \text{ lb./min.}$
$q = \dfrac{60\times 0.185}{0.831} = 13.4 \text{ lb./min.}$	Equation (62)	$q = \dfrac{60\times 0.914}{1.058} = 51.8 \text{ lb./min.}$

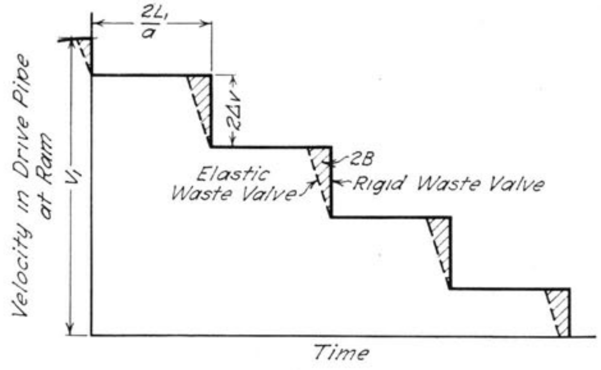

FIG. 20. ASSUMED RELATIONS OF VELOCITY IN THE DRIVE PIPE AT RAM TO TIME DURING PERIOD 3

It is evident from the foregoing examples that evaluation of q_s by the use of Equation (34) is rather tedious. However, in many cases the simplified form given in Equation (43) is sufficiently accurate. Some idea of when the simplified form is permissible may be obtained by noting that if the waste valve is assumed incompressible, the velocity–time curve during Period 3 becomes a series of vertical and horizontal lines, as shown in Fig. 20. In the analysis of Period 3, (Section 11) it has been noted that the area under this curve is proportional to q_s. The difference in area between the two curves for rigid and elastic waste valves is therefore proportional to the error (which is always positive) introduced by considering the waste valve to be rigid. Thus, approximately (see Fig. 20)

$$\text{error in } q_s = 2NwAB \text{ pounds.}$$

But, from Equations (28) and (20),

$$B = v_1 Z \log_e \frac{v_1}{v_2} - Z\Delta v = Z \text{ (function of } v_1 \text{ and } \Delta v).$$

Substitution of this value of B in the equation gives

$$\text{error in } q_s = 2NwAZ \text{ (function of } v_1 \text{ and } \Delta v).$$

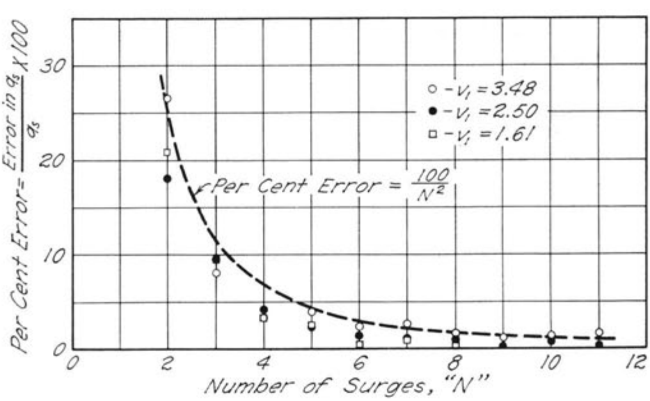

FIG. 21. RELATION OF PERCENTAGE ERROR IN q_s CAUSED BY USING EQUATION (43) TO NUMBER OF SURGES DURING PERIOD 3 FOR THE 4-INCH RAM

$$\text{But percentage error in } q_s = \frac{\text{error in } q_s}{q_s} \times 100$$

$$= \frac{2NwAZ \text{ (function of } v_1 \text{ and } \Delta v)}{\dfrac{2L_1Aw}{a}(Nv_1 - N^2\Delta v)} \times 100,$$

$$\text{or percentage error in } q_s = \frac{aZ}{L_1} \text{ (function of } N,\ v_1, \text{ and } \Delta v).$$

For the 4-in. ram the percentage errors have been computed, and the results plotted in Fig. 21 with percentage error as ordinates and N as abscissa. It appears that changes in Δv or v_1 affect the percentage error much less than changes in N, so that the foregoing equation may be written

$$\text{percentage error} = \frac{aZ}{L_1} \text{ (function of } N).$$

The value of $\frac{aZ}{L_1}$ for the 4-in. ram is approximately 2; therefore the function of N may be approximated by the expression $50/N^2$. Then, for any ram,

$$\text{percentage error} = \frac{50aZ}{L_1N^2}.$$

The criterion for applicability of the simplified equation, Equation (43), may be expressed as

$$N \geq \frac{50aZ}{L_1 \text{ (allowable per cent error)}}.$$

For instance, if the allowable error is 5 per cent,

$$N \geq \frac{50 \times 4380 \times .0264}{56.5 \times 5} = 4.52$$

for the 4-in. ram, and the conclusion is drawn that for the 4-in. ram, Equation (43) may be used to calculate q_s, with an error of less than 5 per cent if N is equal to or greater than 5.

21. *Agreement of Results.*—Results, computed by the foregoing method, show that, for all test ranges of operation, the quantity of water pumped per cycle plotted as a function of delivery head becomes a series of sensibly straight lines, intersecting at the values of delivery head at which N, the number of surges in Period 3, changes. Figure 22, on which both computed and experimental values are plotted, shows a good agreement between the values of delivery head at which the computed changes in slope occur, and the pressures at which the actual changes in slope occur.

For the 2-in. ram, the computed value of the quantity pumped per cycle is generally in agreement with experimental results within ten per cent for a range of pressures from the minimum operating pressure of the ram to an upper limit which increases with increasing values of v_0, the velocity required to begin waste valve closure, and which is generally above half the maximum operating pressure for any value of v_0. Above this limit the computed value of quantity pumped per cycle is greater than the experimental value. The conclusion is drawn that the analysis neglects to take into account an energy loss which becomes important at high delivery pressures. One possible explanation of such loss is found in the nature of the setting of this ram. At high pressures, when the waste valve closed, a movement of the entire ram and drive pipe took place, of the order of an eighth of an inch. This was made possible by the flexibility of the sheet metal tank to which the supply end of the drive pipe was attached and by the ram not being attached to the floor

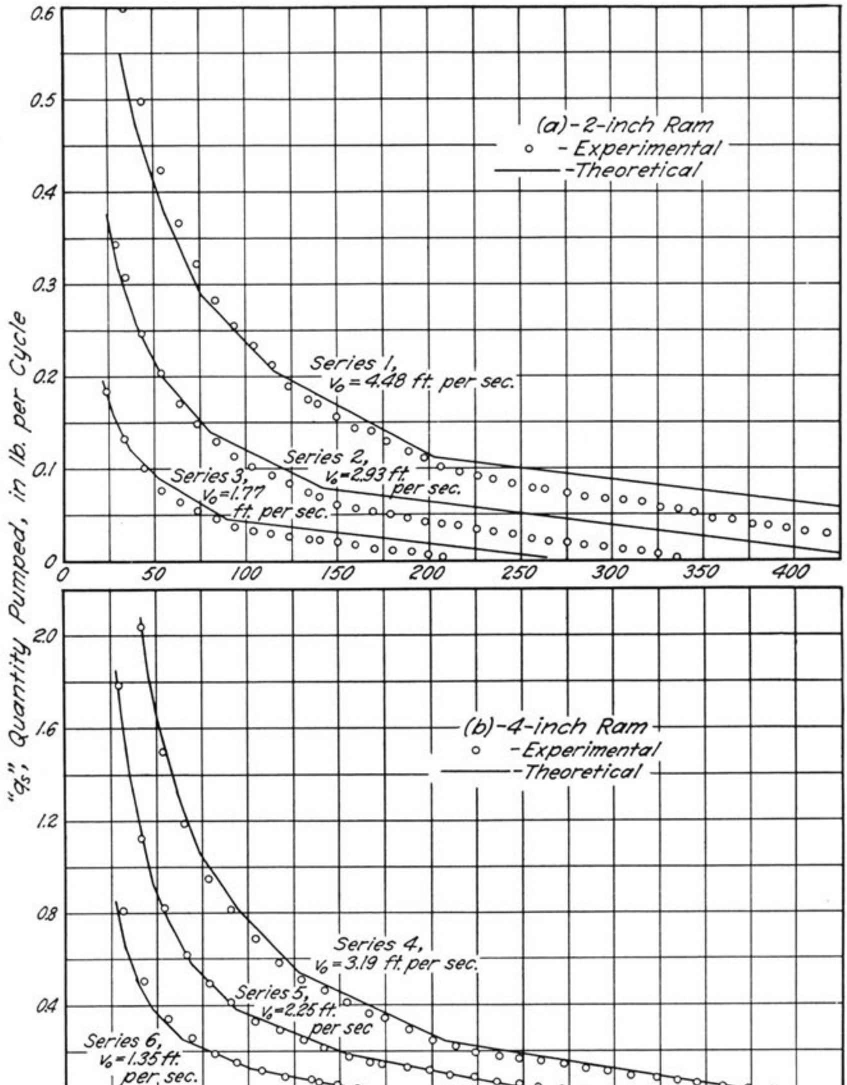

FIG. 22. COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONS OF q_s, THE QUANTITY PUMPED PER CYCLE, TO h, THE STATIC DELIVERY HEAD, FOR THE 2-INCH RAM AND THE 4-INCH RAM

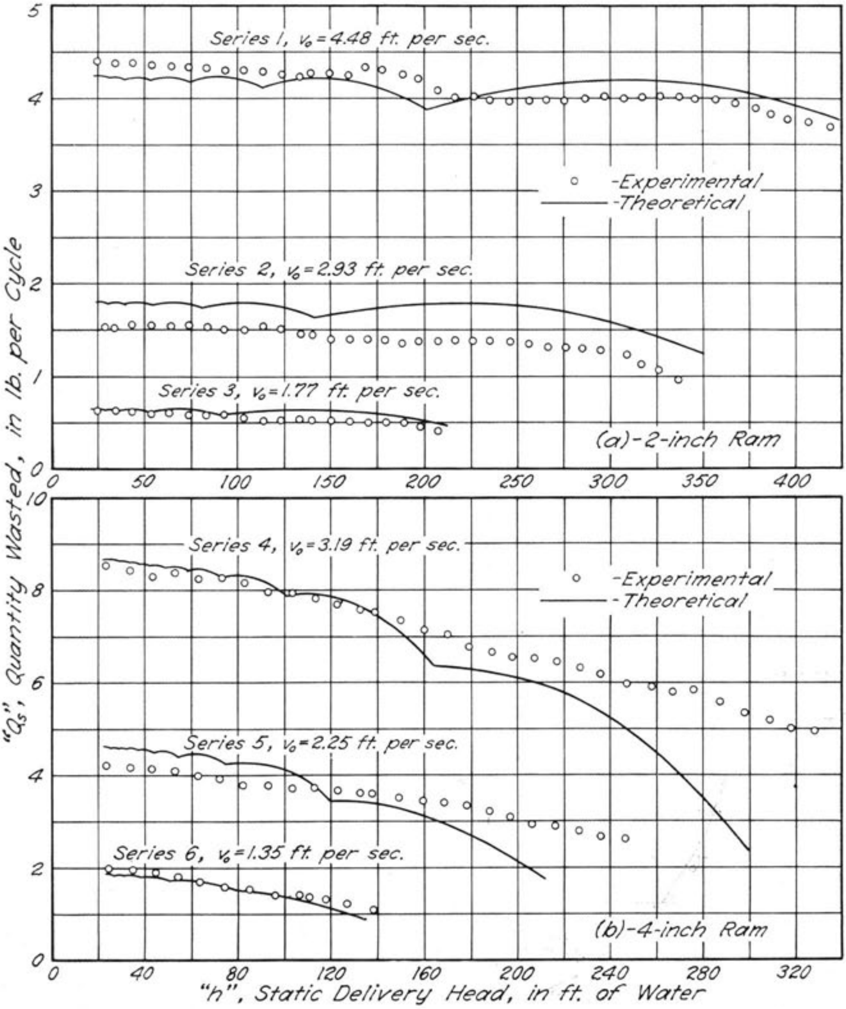

FIG. 23. COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONS OF Q_s, THE QUANTITY WASTED PER CYCLE, TO h, THE STATIC DELIVERY HEAD, FOR THE 2-INCH RAM AND THE 4-INCH RAM

of the laboratory. A considerable amount of energy is doubtless dissipated in such motion, but the exact amount is difficult to determine.

For the 4-in. ram the agreement is good throughout the entire range of the tests.

On Fig. 23 are shown the results of computations of the quan-

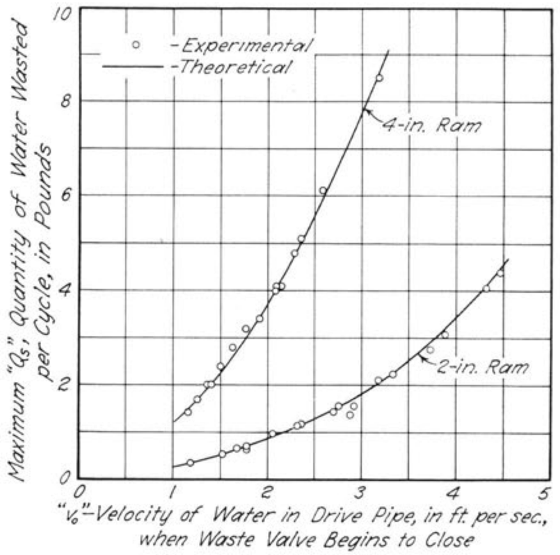

FIG. 24.—COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONS OF MAXIMUM Q_s, THE QUANTITY WASTED PER CYCLE, TO v_0, THE VELOCITY IN THE DRIVE PIPE REQUIRED TO BEGIN WASTE VALVE CLOSURE

tity wasted per cycle for the conditions of the six series of tests. It is evident that the general trends of the curves are similar. For the 4-in. ram the curves slope too steeply where the pressure is high; this indicates that the computed values of v_4 or v_5 are too large, since by Equation (58) it is seen that a large value of v_5 results in a small value of Q_6, and therefore a small value of Q_s (Equation (60)). Probably the assumption that all the energy in the waste valve is returned as the valve expands during Period 4 is responsible for making the computed values of v_4 too large, since the assumption is not true even when the loads are applied slowly (see Fig. 7), and is probably even farther from the truth under faster rates of loading and unloading such as occur in operation of the ram. For the 2-in. ram, a fairly close parallelism, which is most noticeable for the highest value of v_0, is maintained over the entire operating range of the ram.

It is apparent that if the curves were extrapolated to intersect the vertical axis, the intercepts of the computed curves would not all coincide with the values of maximum Q_s which the experimental curves seem to approach. Figure 24 shows the relation of these intercepts for a number of different values of v_0 for which tests were

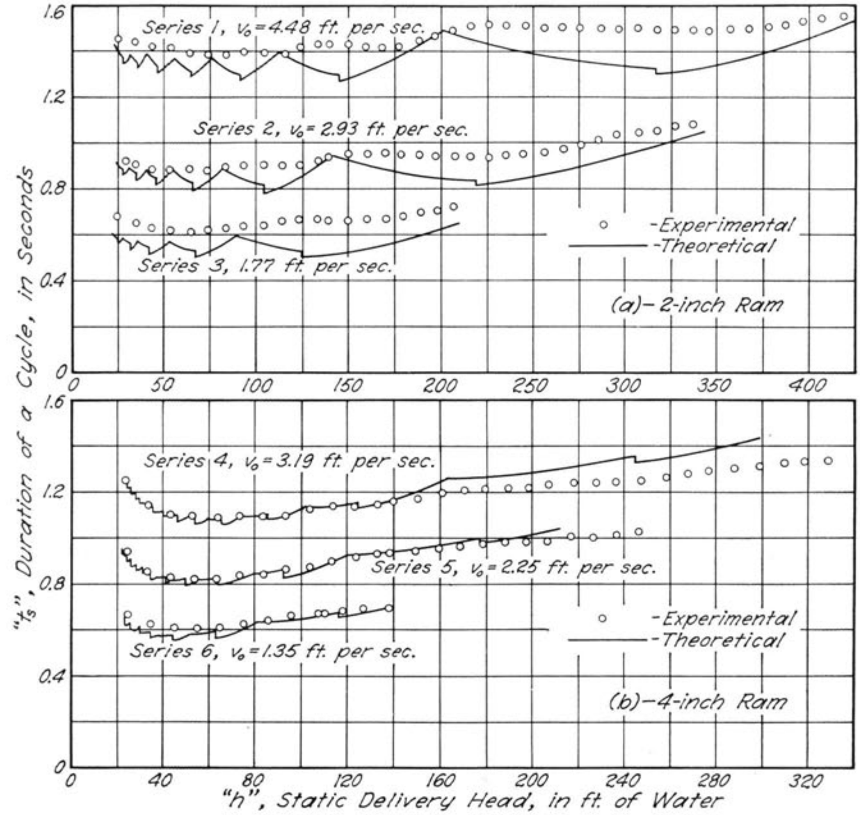

FIG. 25. COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONS OF t_s, THE DURATION OF A CYCLE, TO h, THE STATIC DELIVERY HEAD, FOR THE 2-INCH RAM AND THE 4-INCH RAM

made. From Fig. 24 it may be seen that the method of computing Q_s gives rather good results in general, but that for a few values of v_0 the maximum (intercept) experimental value of Q_s seems a little erratic. It is probable that this is due to inaccurate determination of v_0, which would explain some of the discrepancies noted in connection with Fig. 23.

On Fig. 25 are shown the results of computations for t_s, the duration of one cycle, for the conditions of the six series of tests. Again the general trends of the curves are similar. For the 2-in. ram the experimental points do not follow the dips in the computed curves. The dips in the computed curves are due to the value of v_4 being a minimum at those values of delivery pressure at which

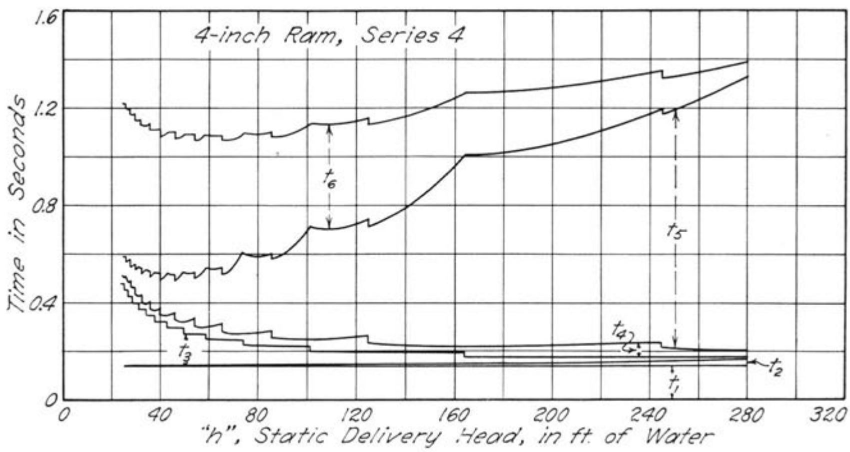

FIG. 26. VARIATIONS OF t_1, t_2, t_3, t_4, t_5, AND t_6, THE DURATIONS OF PERIODS 1 TO 6, INCLUSIVE, WITH h, THE STATIC DELIVERY HEAD, SERIES 4, 4-INCH RAM

v_3 is nearly zero (see Equations (50) and (44)). When the value of v_4 is small, the duration of Period 5 is short (see Equation (54)). Therefore it is probable that v_4 is actually greater than the computed value, for these values of the delivery pressure. The extra energy needed to produce a larger v_4 may come from the recoil from the aforementioned longitudinal movement of the entire apparatus upon waste valve closure.

For the 4-in. ram, considerably better agreement is found, except at very high delivery pressures, where, as previously stated, all the energy is not regained from the waste valve as assumed, which makes the computed values of v_4 and t_5 too great.

In Fig. 26 one of the curves for t_s is broken up into its six parts, to show the variation of each with h.

In Fig. 27 computed curves are added to the experimental data of Fig. 17, to show correspondence of the variation in the several curves.

V. CONCLUSIONS

22. *Summary and Conclusions.*—It was the purpose of this investigation to make a rational mathematical analysis of the operation of single-acting hydraulic rams and to compare the results of such an analysis with those obtained from experimental data. It was hoped that the analysis would make it possible to calculate satisfactorily the useful characteristics of the ram, especially the

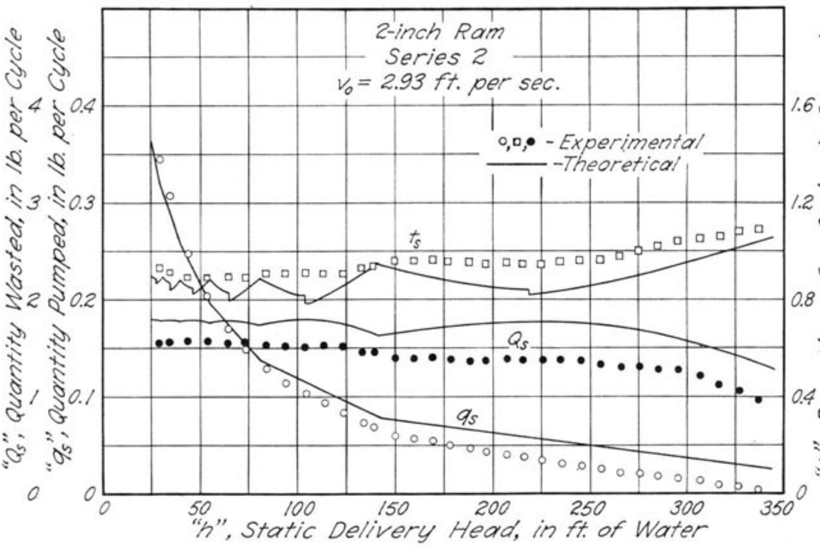

FIG. 27. COMPARISON OF THEORETICAL AND EXPERIMENTAL RELATIONS OF q_s, THE QUANTITY PUMPED PER CYCLE, Q_s, THE QUANTITY WASTED PER CYCLE, AND t_s, THE DURATION OF A CYCLE, TO h, THE STATIC DELIVERY HEAD, SERIES 2, 2-INCH RAM

rate of pumping and of wasting for a given supply head, delivery pressure, and waste valve setting. The analysis has been made and is rational except for some minor assumptions which are based on experimental evidence.

Experimental data have been taken on two rams, one having a supply or drive pipe of constant diameter whose nominal value was four inches (designated as a 4-in. ram); the other was a 2-in. ram. The 4-in. ram had a soft waste valve disc, whereas the 2-in. ram had a very hard or rigid waste valve disc. The drive pipe for each ram had a slope of about one ft. in 25 ft. In the mathematical analysis the drive pipe was assumed to be horizontal, since the slope of the drive pipe does not affect the results of the analysis. The dominant factor controlling the functioning of the ram is v_0, the velocity in the drive pipe necessary to cause the waste valve to start closing, and its value is fixed by the waste valve setting. For each ram, series of tests were made with three different values of v_0. In each series of tests the delivery pressure was varied in approximately equal increments over the entire range of pressures under which the ram would continue to operate, all other factors being held constant. The quantity pumped, the quantity wasted, the supply head, the delivery pressure, and the time of the cycle were carefully determined for all series of tests.

A comparison of the results of the mathematical analysis with the results of the experimental data taken from the two rams of different sizes and considerably different characteristics for values of the velocity in the drive pipe covering a wide range of operation appears to justify the following conclusions:

1. (a) The quantity pumped *per cycle* and the quantity wasted *per cycle* can be predicted with an error less than 10 per cent for delivery pressures less than one-half the maximum pressure the ram is capable of developing for a particular setting of the waste valve. The analytical and experimental results are shown graphically in Figs. 22 and 23. For delivery pressures greater than one-half the maximum obtainable, the quantity pumped and quantity wasted may be predicted with somewhat greater error, as indicated in Figs. 22 and 23. However, at these higher delivery pressures the quantity pumped is so small that in general the use of the ram under such conditions is not practical. The correlation between the mathematical and the experimental results is better for the larger ram, particularly for the higher delivery pressures.

(b) The quantity pumped and the quantity wasted *per unit of time* may be computed from the quantities pumped and wasted per cycle if the length of time required for a cycle can be determined. This time for a cycle may be computed with an error less than 10 per cent for the 4-in. ram and less than 20 per cent for the 2-in. ram, as is indicated in Fig. 25.

2. The analysis was based on the fact that pumping was performed by several rapid pressure waves or impulses of the water in the drive pipe during the pumping period in each cycle (designated as Period 3 in the analysis), and not by one surge as the sound of the operating ram would seem to indicate. These pressure waves are indicated on the records of the oscillograph in Fig. 16. This phenomenon is also revealed by the series of straight-line portions of the curves that show the relation between the quantity pumped per cycle and the delivery pressure, as indicated in Fig. 11.

3. In the mathematical analysis involving the pressure–wave phenomenon, the elasticity of the waste valve cannot be neglected for these rams as is evidenced by the marked difference in the performance curves for the two rams as shown in Figs. 11 to 13. (The 2-in. ram was equipped with a much stiffer valve than was the 4-in. ram.)

4. Some of the more important operating characteristics of the single-acting hydraulic rams tested were found to be as follows:

(a) An increase in the delivery pressure or head decreased the quantity pumped per cycle, as indicated in Fig. 11.

(b) An increase in the delivery head decreased the quantity wasted per cycle as indicated in Fig. 12. This decrease was more pronounced in the larger ram.

(c) The time of the cycle decreased as the delivery head increased up to a certain value of the delivery head, (approximately 50 ft. for the 4-in. ram, and 75 ft. for the 2-in. ram); at this head the time of the cycle was a minimum. Further increase in the delivery head increased the time of the cycle as shown in Fig. 13.

(d) The delivery head for which maximum efficiency was attained increased as v_0 increased, v_0 being the velocity in the drive pipe necessary to begin closure of the waste valve.

(e) The maximum efficiency attained varied little with various adjustments of the waste valve, except perhaps for extremely high values of v_0, at which the efficiency was somewhat lower.

APPENDIX

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