How to Compute Forebay Storage

The author presents a simple method of computing forebay storage, regarding the problem as a two-dimensional steady-flow condition with a time variable

By EDWARD J. LOW*

IF the power drop is situated at the end of a headrace canal, the forebay serves essentially as a regulating reservoir at the head of the penstocks. The purpose of a forebay is to store water temporarily when the load on the plant is reduced or to supply water for an increasing load while water in the headrace is being accelerated.

To find the capacity of the forebay, it is customary to take the maximum discharge under uniform-flow conditions at the headrace multiplied by t seconds. If t is the length of time that the demand of the turbine is being supplied by the forebay storage, and if we assume a full reduction or increase of load, the temporary storage will be:

$$S = tQ \qquad \text{... (1)}$$

where S is the storage in ft^3, t is the time in seconds, and Q is the discharge of the canal in cusecs. If the storage is capable of taking care of only 50% of the discharge in one hour then the storage is given by:

$$S = 3{,}600 \times 0{\cdot}50,\ Q = 1{,}800\ Q \qquad \text{... (2)}$$

We should observe that any reduction or increase of the turbine demand will cause a non-uniform flow condition in the headrace. If the intake level is steady, the water in the headrace canal will have a backwater profile or a drop-down profile, depending upon the level of the forebay. Thus, the discharge of the canal is less while the water is being accelerated.

In order to calculate the deficiency of the delivery, we must calculate the velocity of the moving water at any instant during the acceleration. Since this is a problem of unsteady gradually varied flow of water in open channel, we may consider the surface to be nearly straight. If the mass does not change with time, the continuity and dynamic equations may be derived in accordance with Newton's law of motion. This will lead to the solution of unsteady flow in open channels.

Although the continuity and the dynamic equations for unsteady flow have been published by Saint-Venant during the past century, owing to their mathematical complexity, exact solutions of these partial differential equations is impossible. For practical engineering applications, simplified assumptions may be used to obtain a solution.

The following derivation assumes the channel to be prismatic and the change in velocity head and depth from station to station to be very small. The flow area also remains approximately constant with no upward acceleration.

Let us consider this to be a two-dimensional steady-flow problem with a time variable. This time variable takes into effect the change in velocity of the moving water due to the gravity force against the retarding force produced by the friction between the moving water and the bed, which varies with the square of the velocity. Referring to Fig. 1, the problem of gradually unsteady flow can be approached readily by Bernoulli's theorem by considering the concept of energy loss, which applies equally to steady flow, where

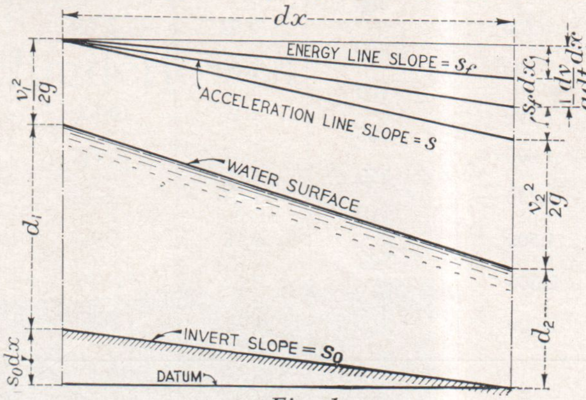

Fig. 1

the head loss due to acceleration is expressed by $\frac{1}{g}\frac{dv}{dt}dx$. As indicated in the diagram, the Bernoulli equation can be written as follows for open channels between two sections a distance dx apart.

$$s_o\ dx + d_1 + \frac{v_1^2}{2g} = d_2 + \frac{v_2^2}{2g} + s_f\ dx + \frac{1}{g}\frac{dv}{dt}\ dx \qquad \text{... (3)}$$

Simplifying according to the foregoing assumptions we have:

$$\frac{1}{g}\frac{dv}{dt} = s_o - s_f \qquad \text{... (4)}$$

Noting that:

$$s_o = k_1\ v_n^2 \text{ and } s_f = k_2\ v^2$$

where v_n^2 is the design velocity as determined by the Manning formula.

According to the law of quadratic resistance, if k_1 and k_2 are approximately equal. Equation (4) can then be written as:

$$\frac{dv}{dt} = \frac{s_o\ g}{v_n^2}\ (v_n^2 - v^2) \qquad \text{... (5)}$$

This is the dynamic equation for accelerating flow in a given channel. By rearranging the equation and integrating we obtain:

$$v = v_n \tanh\left[\left(\frac{s_o\ g}{v_n}\right)t + C\right] \qquad \text{... (6)}$$

where C is an integration constant which may be determined by the initial condition. At the inlet, $t=0$ and $v=0$. Thus $C=0$.

Let

$$c = \frac{s_o\ g}{v_n}$$

then

$$v = (\tanh\ ct)\ v_n \qquad \text{... (7)}$$

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Table I

Time t	Δt	ct	tanh ct	v	Flow = 167·4 v	Average flow	Volume delivered
sec				ft/sec	cusecs	cusecs	ft³
0		0·00	0·000	0·000	000·0		
	100					25·8	2,580
100		0·13	0·129	0·308	51·6		
	100					76·6	7,660
200		0·26	0·254	0·607	101·6		
	100					126·8	12,680
300		0·40	0·380	0·908	152·0		
	100					173·0	17,300
400		0·53	0·485	1·159	194·0		
	100					212·6	21,260
500		0·66	0·578	1·381	231·2		
	100					247·3	24,730
600		0·79	0·658	1·573	263·3		
	100					276·9	27,690
700		0·92	0·726	1·735	290·4		
	100					302·5	30,250
800		1·06	0·786	1·879	314·5		
	100					323·5	32,350
900		1·19	0·831	1·986	332·5		
	100					339·7	33,970
1,000		1·32	0·867	2·072	346·9		
	250					359·3	89,825
1,250		1·65	0·929	2·220	371·6		
	250					378·5	94,625
1,500		1·98	0·963	2·302	385·4		
	500					390·8	195,400
2,000		2·64	0·990	2·366	396·1		
	1,000					398·0	398,000
3,000		3·96	0·999	2·389	399·9		
Σ	3,000						988,320

Volume required = 3,000 × 400 = 1,200,000 ft³.
Storage = volume required–volume delivered = 1,200,000– 988,320 = 211,680 ft³.

It is interesting to note that the velocity of the water in the channel approaches the normal velocity asymptotically. This follows the quadratic resistance law, in which both the time and distance are infinite.

Equation (7) may be used to approximate the velocity of the moving water at any time and thus estimate the volume of the water delivered to the forebay during the acceleration.

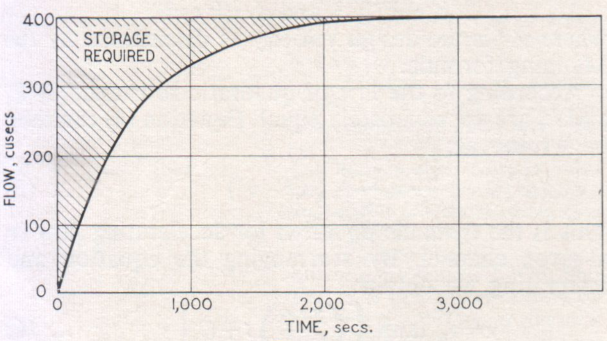

Fig. 2

Example:

A hydro-electric installation has a design discharge of 400 cusecs. Water is being diverted to the power house by a rectangular headrace channel 18·30 ft in width and a mile in length. The channel invert slope is 0·000098. The Manning roughness coefficient n is 0·017. Determine the storage required to provide a sufficient supply to the forebay at the head of the penstocks, so that a sudden collapse of the forebay level and admission of air into the penstocks and turbines, due to a full increase of load in the power plant, will not occur.

Solution:

Design velocity $v_n = 2·39$ ft/sec for hydraulic radius $= \frac{d}{2}$, $d = 9·15$ ft, $b = 18·3$ ft, and flow area $A =$ 167·4 ft².

$$c = \frac{s_o\ g}{v_n} = \frac{0·000098 \times 32·2}{2·39} = 0·00132$$

The values can now be worked out as in Table I.

Conclusion

The final determination of the amount of storage in the forebay depends on good judgment of economical considerations. Occasionally, a forebay is formed by building a dam across a natural-flow stream channel, where ample storage can be obtained at a very low cost compared with the power-plant structures. A reduction in the flow through the turbines, however, will cause the forebay level to rise and the head to increase, and promote further reduction of flow. Likewise, increase in flow will cause the forebay level to drop and the head to decrease, and promote further increase of flow. The level and storage capacity of the forebay have much to do with the dynamic response of the generating unit of a hydro-electric plant, because of the large inertia of the moving water. An economic analysis of such an operation is quite involved and very difficult; the choice of the forebay water-surface area and storage has been mainly empirical.

NOTATION

A = area of cross section—ft^2.
c = a constant.
C = an integration constant.
d = approximate depth of water if the slope of the channel is very small—ft.
g = acceleration due to gravity.
k = proportional coefficient.
n = roughness coefficient in Manning's formula.
Q = discharge—cusecs.
s = acceleration line slope.
s_o = invert slope.
s_f = energy line slope.
t = time—sec.
v = mean velocity of flow at any time—ft/sec.
v_n = design velocity as determined by the Manning's formula—ft/sec.
x = distance—ft.

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Progress at Hunderfossen

Hunderfossen dam under construction

The accompanying photograph shows work in progress on the Hunderfossen dam in Norway. The dam, which is situated on the Gudbrandsdalslågen river, is to be 985 ft long and 40 ft high. Two tunnels will supply two generating sets, each consisting of a 58,000 kVA generator and a 70,000 h.p. Kaplan turbine, running at 167 r.p.m. under a head of approximately 136 ft. The units are to be housed in an underground machine hall which will be 246 ft long, 50 ft wide and 115 ft high. It is thought that the 2½-mile horseshoe-section tailrace tunnel, which passes underneath the river, will be the largest tunnel to be driven in Norway. Because of the tunnel's large area (1,400 ft^2), it is being driven in two sections. The full length of the top half is now nearing completion. It is driven from three places and a 500 g.p.m. influx of water has made it necessary to install large capacity pumps to keep the working faces dry. The rock consists of sparagmite and schist, and zones of disturbed rock have called for extensive roof bolting and concrete lining. Excellent progress has been maintained and the best week's performance in a heading so far has been an advance of 144 ft in the 732-ft^2 tunnel in 17 eight-hour shifts. Each of the daily shifts consisted of nine men with eight airleg-mounted Holman Silver Three rockdrills operating on the face. A 105-hole drilling pattern gave an average pull of 11·15 ft over 13 rounds. About half the compressed air used on site is supplied by six Holman T60R compressors, each giving a free-air delivery of 525 ft^3/min at 100 lb/in^2. Holman Silver Three drills have been used exclusively by the tunnelling contractors, Ing. Thor Furuholmen A-S, of Oslo. Work on the project started in November 1960 and is due to end late in 1963.