Specific Energy in Circular Channels

The author replies to criticisms of an earlier article on this subject contributed to this journal

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IN a recent article, Jenkner[1] proposed a method for computing the relationship between the critical depth, the discharge and the minimum specific energy in circular channels, criticising an earlier paper by the writer[2] for not providing the relationship between the last quantity and the first two.

The main reason for not providing this information earlier is the great ease with which this relationship can be obtained from published tables of properties of parts of a circle, such as those given by King[3] or Chow[4]. These tables provide, among other data, values of the mean hydraulic depth $\overline{h}=A/B$, in a non-dimensional form $\overline{h}/D$, for values of relative depth h/D in the channel at 0·01 intervals of h/D. The minimum specific energy $H_{\min}$ for any given critical depth h_c can be obtained from the above tables simply by using the equation:

$$\frac{H_{\min}}{D}=\frac{h_c}{D}+\frac{1}{2}\frac{\overline{h_c}}{D} \quad \ldots (1)$$

and reading from the tables the corresponding values of h/D and $\overline{h}/D$. If on the other hand the reverse problem is presented, that of determining the critical depth for a given minimum specific energy, the same equation (1) could be solved by expressing the minimum specific energy non-dimensionally as $H_{\min}/D$ and selecting by successive approximation a value of h_c/D from the tables, which satisfies equation (1).

As an aid for the solution a table and a graph could be prepared, from Chow's table for example, giving values of the minimum specific energy for various values of h_c/D. Such a table and a graph (Table I and Fig. 1) would naturally be similar to those produced by Jenkner[1]; his equations are of course an explicit development of the geometrical relationships which were, no doubt, used by the compilers of the tables of properties of parts of a circle mentioned above. It should be noted also that the second graph, $h_c/r\delta$ versus h_c/r, produced by Jenkner, is, with a slight change of variables, similar to that of $Q/(gD^5)^{\frac{1}{2}}$ versus (h_c/D), produced by the writer[2] in 1958, since:

$$\frac{a}{\delta}=\frac{h_c}{r\delta}=\left(\frac{Q^2}{gr^5}\right)^{1/3}=2^{5/3}\left[\frac{Q}{(gD^5)^{\frac{1}{2}}}\right]^{2/3} \quad \ldots (2)$$

TABLE I

$\frac{h_c}{D}$	$\frac{H_{\min}}{D}$	$\frac{h_c}{D}$	$\frac{H_{\min}}{D}$
0·00	0·0000	0·50	0·6964
0·05	0·0668	0·55	0·7724
0·10	0·1341	0·60	0·8511
0·15	0·2017	0·65	0·9333
0·20	0·2699	0·70	1·0204
0·25	0·3387	0·75	1·1148
0·30	0·4036	0·80	1·2210
0·35	0·4784	0·85	1·3482
0·40	0·5497	0·90	1·5204
0·45	0·6223	0·95	1·8341
		1·00	∞

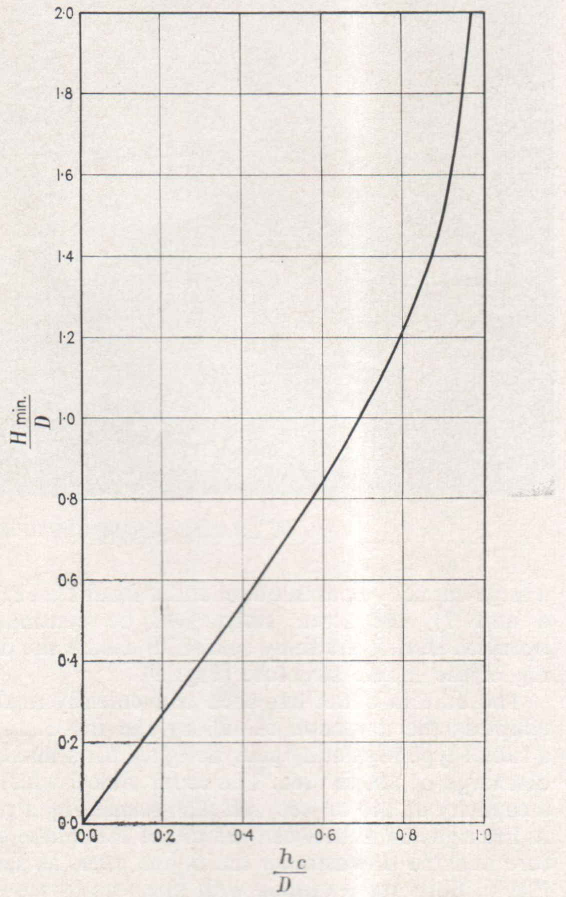

Fig. 1

The advantage claimed by Jenkner for his solution, that of direct computation, is of course lost once use is made of the graph or table provided by him for the practical solution.

The usefulness of the tables of properties of parts of a circle mentioned above could be further demonstrated with reference to the more general problem of computing the relationship between specific energy H, depth of flow h, and discharge Q in circular channels, when conditions are not critical. The equation relating these quantities and defining the specific energy, based on the usual assumption of uniform velocity distribution and no curvature of flow, is:

$$H=h+\frac{V^2}{2g}=h+\frac{Q^2}{2gA^2} \quad \ldots (3)$$

Using the diameter of the channel D as denominator

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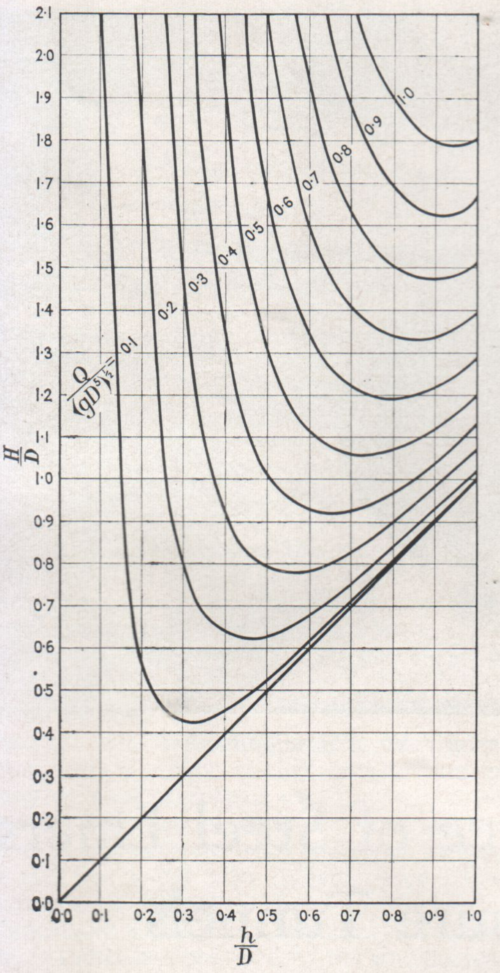

Fig. 2

for length ratios, this equation can be expressed non-dimensionally as:

$$\frac{H}{D}=\frac{h}{D}+\frac{Q^2/gD^5}{2(A/D^2)^2} \qquad \dots (4)$$

Here again, while it is possible to substitute for A/D^2 in terms of h/D to get an explicit expression, it is much simpler to use the tables of parts of a circle. Corresponding values of h/D and A/D can be taken from the tables and used directly in equation (4) if H or Q is the unknown, or the values can be used as successive approximations if h is the unknown.

A table of H/D values for various assumed values of $Q/(gD^5)^{\frac{1}{2}}$ and h/D (Table II) and a diagram giving the relationship between the three variables (Fig. 2), have been prepared[5] by using equation (4). These can be used as a guide to a more accurate solution of the equation, or if the accuracy required does not justify refinements, values of the unknown variable can be obtained directly from the table and graphs by suitable interpolations.

Acknowledgment

Table II and Fig. 2 were reproduced from a thesis prepared by the writer under the direction of Professor S. Davis at the Technion, Israel Institute of Technology.

NOMENCLATURE

a=non-dimensional critical depth, $a=h_c/r$.
A=cross-sectional area of flow.
B=width of channel at water surface.
c=subscript referring to critical section.
D=diameter of circular channel.
g=acceleration due to gravity.
h=depth of flow.
$\bar{h}$=mean hydraulic depth, $\bar{h}=A/B$.
H=specific energy.
H_{min}=minimum specific energy.
Q=discharge.
r=radius of circular channel, $r=D/2$.
V=mean velocity of flow, $V=Q/A$.
δ=factor defined by Jenkner[1].

TABLE II

h/D	Values of H/D for $Q/(gD^5)^{\frac{1}{2}}=$									
	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0·00	∞									
0·05	23·2									
0·10	3·089									
0·15	1·066	3·812								
0·20	0·600	1·800	3·800							
0·25	0·462	1·099	2·160	3·645						
0·30	0·427	0·809	1·446	2·336	3·482					
0·35	0·433	0·683	1·100	1·683	2·432	3·339				
0·40	0·461	0·644	0·948	1·375	1·852	2·486	3·242			
0·45	0·493	0·620	0·833	1·130	1·514	1·981	2·535	3·173		
0·50	0·532	0·630	0·792	1·019	1·310	1·667	2·089	2·575	3·126	
0·55	0·576	0·652	0·780	0·958	1·188	1·469	1·800	2·183	2·617	3·102
0·60	0·621	0·683	0·786	0·931	1·116	1·344	1·612	1·922	2·274	2·666
0·65	0·667	0·718	0·804	0·924	1·078	1·266	1·489	1·746	2·037	2·462
0·70	0·714	0·758	0·830	0·932	1·062	1·222	1·410	1·628	1·874	2·150
0·75	0·763	0·800	0·863	0·950	1·063	1·201	1·364	1·552	1·765	2·003
0·80	0·811	0·844	0·899	0·976	1·076	1·197	1·340	1·505	1·693	1·902
0·85	0·860	0·890	0·939	1·008	1·097	1·206	1·334	1·482	1·650	1·838
0·90	0·909	0·936	0·981	1·044	1·126	1·225	1·342	1·477	1·631	1·802
0·95	0·958	0·984	1·026	1·085	1·160	1·253	1·363	1·489	1·632	1·792
1·00	1·008	1·041	1·073	1·130	1·203	1·292	1·397	1·519	1·657	1·811

REFERENCES

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3. KING, H. W., and BRATER, E. F. "Handbook of Hydraulics." McGraw-Hill Book Co., New York, 4th Edition, 1954.
4. CHOW, V. T. "Open Channel Hydraulics." McGraw-Hill Book Co., New York, 1959.
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Weatherproof Lighting Fittings. A leaflet (No. L625) received from Victor Products (Wallsend) Limited, Wallsend-on-Tyne, England, describes flameproof and weatherproof lighting fittings for tubular fluorescent lamps. Lamps are replaced by withdrawal from one end of the fitting which is available in 4-ft, 40-W and 5-ft, 80-W models.

Vee-link Belting. A brochure has been received from R. & J. Dick Limited, Greenhead Works, Glasgow, describing the Dixlink V-belts manufactured by this firm. Data regarding belt sizes and capacities for various applications are given.

Fibreglass Insulation. Fibreglass Superfine acoustic and thermal insulation material, and its applications in many fields, are dealt with in a pamphlet recently published by Fibreglass, St. Helens, Lancashire.