Profile Equation for the Hydraulic Jump

This article presents a generalised equation for the mean flow profile of the hydraulic jump for Froude numbers from 5 to 12, from the results of a systematic experimental investigation

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A HYDRAULIC jump is formed when a supercritical stream of water impinges on a subcritical stream. There is an abrupt rise in the water surface from the lower supercritical to the higher subcritical stage, and a violent roller appears on the surface of the jump. A considerable amount of air is entrained, and considerable energy dissipated.

The beginning or toe of the jump is the section at which there is an abrupt rise in the water surface and is well defined, but there has been no general accord as to the position of the end of the jump. Bakhmeteff and Matzke[1] took the end of the jump at the section where the water surface became almost level. Bradley and Peterka[2] defined the end of the jump either as

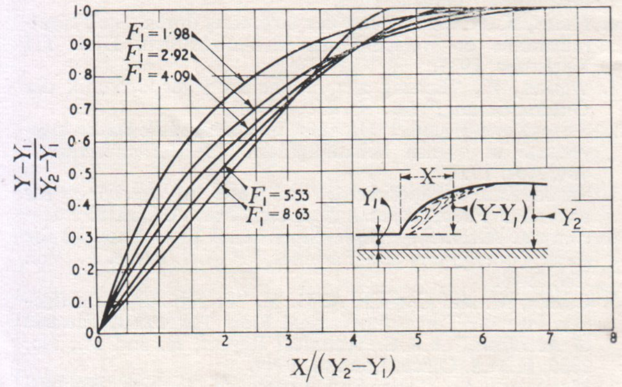

Fig. 1

the end of the surface roller or the section at which the high-velocity jet began to leave the floor, whichever was farther away from the toe. There are a number of other investigations[3] on this aspect and at least a dozen formulae are available[4] for the length of the jump, which is the distance between the toe and the end of the jump.

A precise knowledge of the mean flow profile of the jump would be useful (a) in the economic design of the side walls of stilling basins, (b) for calculating the apron thickness of hydraulic-jump-type basins on permeable foundations, (c) for fixing the flow profile of a closed-conduit model of the jump (as was done by Rouse and others[5]), (d) for evaluating the water-surface roughness produced by friction blocks, (e) for comparing the performance of the block jump and baffle jump with that of the normal jump, (f) to find the effect of velocity distribution of the supercritical stream on the profile and hence on the length of the jump, and (g) also to find the effect of pre-entrainment of air on the profile of the jump.

Bakhmeteff and Matzke[1] gave a dimensionless plot of the profile of the jump formed below a sluice gate, which is reproduced in Fig. 1 from Chow[6]. Moore[7] repeated the same procedure for the jump formed below a free overfall, but to the knowledge of the author, no attempt seems to have been made so far to give a generalised equation for the mean flow profile of the jump. In fact, the need for such an investigation was expressed by Bhandari[8] in 1950. Based on a systematic experimental investigation, such a generalised equation is presented in this article.

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Generalised Equation

Fig. 2 represents the definition sketch for the mean flow profile of the jump. Y_1 and Y_2 are the sequent depths and F_1 is the supercritical Froude number. A co-ordinate system is taken as shown in the figure. To make the profile equation dimensionless, the X and Y co-ordinates are divided by Y_2, the subcritical sequent depth and denoted as α and β respectively. The simplest type of equation chosen for the mean profile is

$$\beta = A_1\ \alpha + A_2\ \alpha^2 \qquad \dots (1)$$

where A_1 and A_2 are constants to be evaluated experimentally. It was believed that A_1 and A_2 might be functions of only F_1, the supercritical Froude number, and this has been very well borne out by the experimental results to be presented below.

Experimental Results

The experiments were conducted in a smooth wooden flume 12 in wide, 15 in high, and 15 ft long. As shown in Fig. 3, water from a constant-head tank entered the flume through a set of transitions with an inlet depth of about 0·95 in. The jump was formed at a distance of about 18 in from the efflux section. The supercritical depth was measured by a precision point gauge and the mean flow profile of the jump as well as the downstream sequent depth were obtained from a series of bottom piezometers located in the bottom of the flume and connected to a manometer board. The discharge was determined by a measuring tank. The investigation covered a range of Froude numbers from 3·50 to 11·30. Dimensionless plots of the profile of the jump for all the eleven

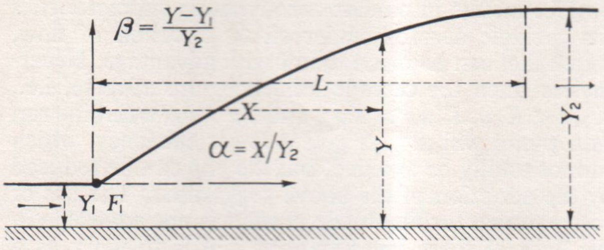

Fig. 2

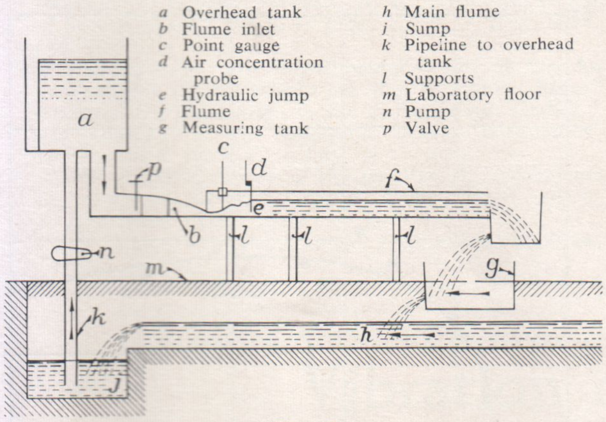

Fig. 3

Froude numbers can be found in the author's thesis[9], and for the sake of brevity only three plots are reproduced in Fig. 4.

The end of the jump was taken as the section at which the surface becomes almost level, that is, at which the depth of flow becomes equal to the subcritical sequent depth. The variation of the length of the jump (based on this criterion) with the Froude number F_1 is shown in Fig. 5. Fig. 5 also contains the curves as per the criteria of Ramamoorthi[10] and Bradley-Peterka[2]. It can be seen that the length of the jump as given by the first criterion is much greater than that given by the other two, especially for the lower Froude numbers; but since the main concern in this article is that of the profile of the jump, only the length given by the first criterion and marked as curve 3 is used for further study.

Evaluation of the Profile Equation

Using the experimental results, the constants A_1 and A_2 are obtained from the solution of the two simultaneous equations

$$A_1 \sum \alpha^2 + A_2 \sum \alpha^3 - \sum \alpha \beta = 0 \qquad \ldots (2)$$

$$A_1 \sum \alpha^3 + A_2 \sum \alpha^4 - \sum \alpha^2 \beta = 0 \qquad \ldots (3)$$

which were obtained from Equation (1) using the principle of least squares. The constants A_1 and A_2 were solved for all the eleven cases and were found to be unique functions of F_1 as shown in Figs. 6 and 7. It was found that the form of the equation

$$\beta = A_1 \alpha + A_2 \alpha^2 \qquad \ldots (1)$$

predicts the profile of the jump satisfactorily for Froude numbers greater than 5·0. For F_1 less than 5·0, various other forms were tried, but without success. It is believed that Equation (1) with a few more terms might solve the difficulty, but the amount of labour involved in such a procedure is enormous and further the resulting formula will be complicated. Hence it was not tried and it is recommended that Equation (1) be used for predicting the profile of the jump for Froude numbers from 5 to 12.

The maximum value of β, i.e. $\beta_{\max}$, occurring at the end of the jump, i.e. at $\alpha_{\max}$, can be shown to be given by the equation

$$\beta_{\max} = \frac{\phi - 1}{\phi} \qquad \ldots (4)$$

where ϕ is the ratio of the sequent depths.

Using the profile Equation (1) with the constants A_1 and A_2 taken from Figs. 6 and 7, the values of β are plotted against α up to $\alpha_{\max}$ (taken from curve 3 in Fig. 5). A smooth curve is drawn through the plotted points, leaving out the points, if any, which

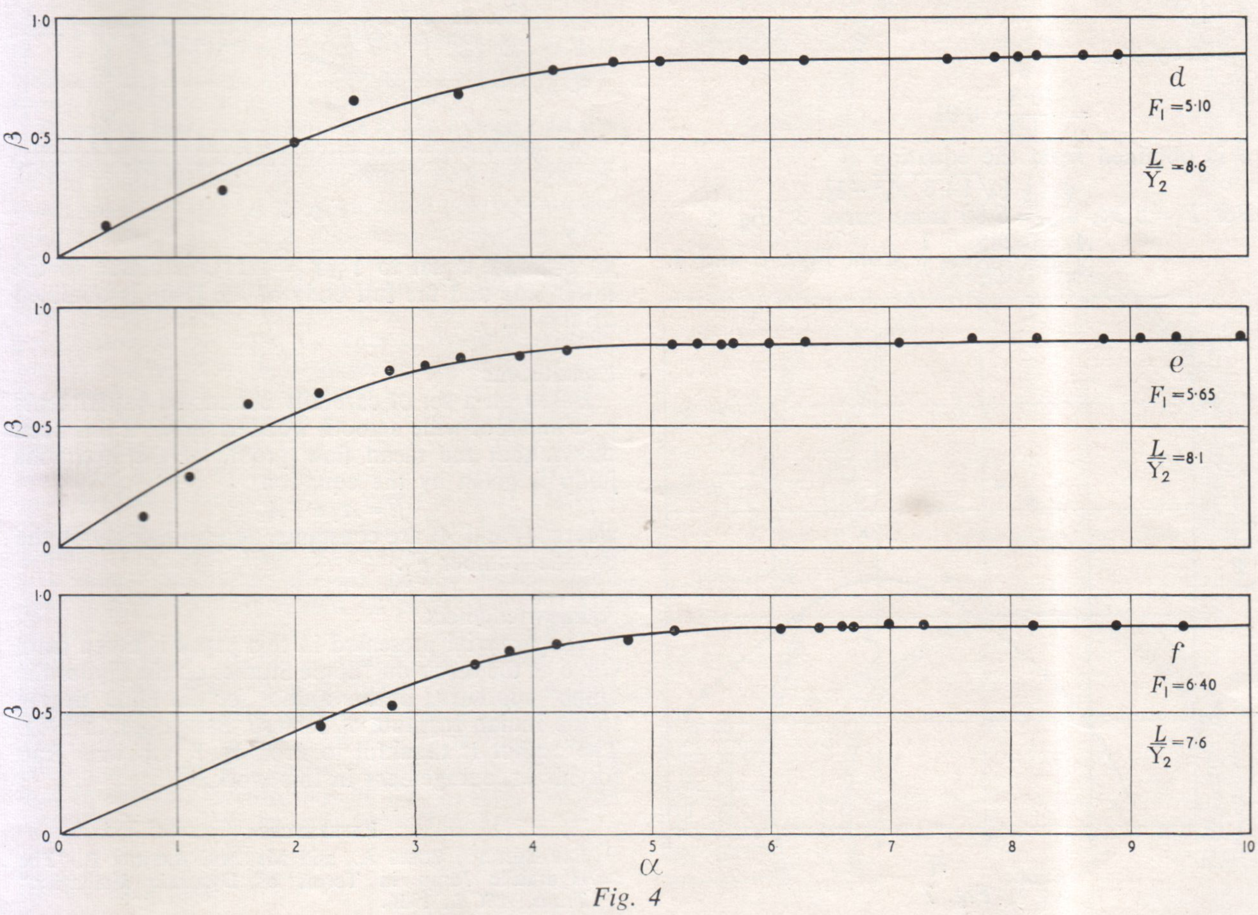

Fig. 4

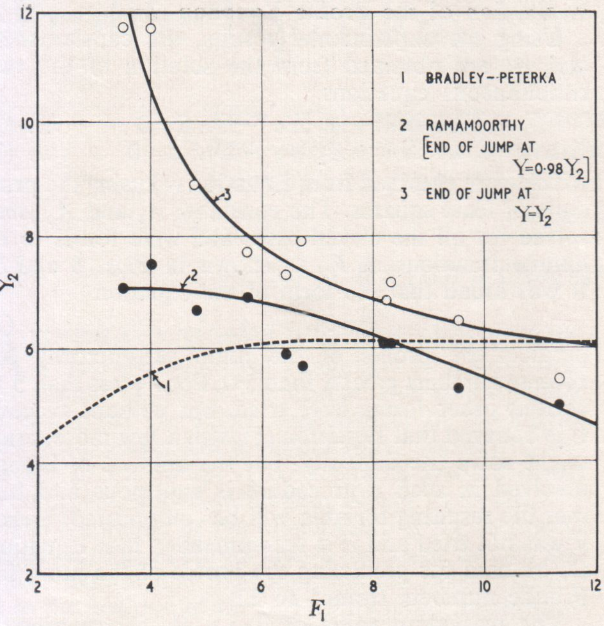

Fig. 5

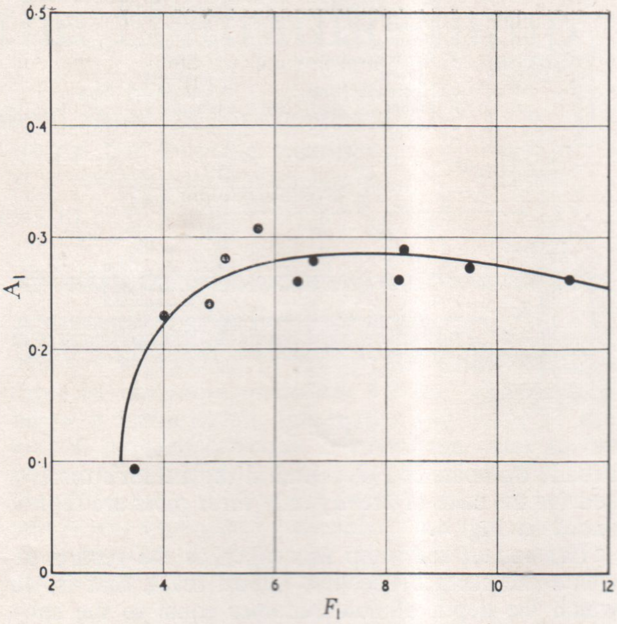

Fig. 6

have β greater than β_{max}. (This seems to be a defect in this method.) Then if an ordinate equal to Y_1/Y_2 is added to the bottom profile, the full body of the jump is obtained, as illustrated in the following example.

EXAMPLE:

Predict the mean flow profile of the jump for $F_1 = 8{\cdot}0$ and $Y_1 = 1{\cdot}0$ ft. Also show the full body of the jump.

Firstly $\beta_{max} = \dfrac{\phi - 1}{\phi}$

$$= \frac{10{\cdot}3}{11{\cdot}3} = 0{\cdot}91$$

ϕ is obtained from the equation

$$\phi = \tfrac{1}{2}\ [\sqrt{1 + 8\ F_1^{\,2}} - 1]$$

For $F_1 = 8{\cdot}30$, $\alpha_{max} = 6{\cdot}80$ from curve 3, Fig. 5

$$\left.\begin{array}{l} A_1 = 0{\cdot}285 \\ A_2 = -0{\cdot}0222 \end{array}\right\} \text{ from Figs. 6 and 7.}$$

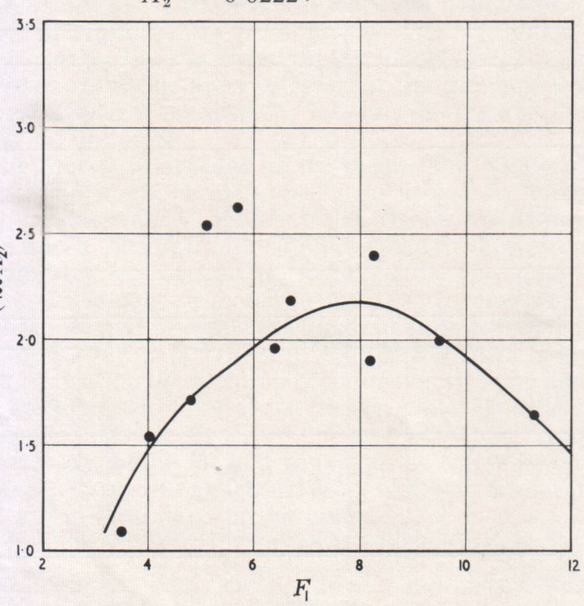

Fig. 7

Using the profile equation and the calculated coefficients, values of β are computed for various values of α and plotted as shown in Fig. 8. A smooth curve drawn through them gives the mean flow profile of the jump. (The black circles shown in the figure are the experimental points for a case with the same Froude number.) To get the full body of the jump,

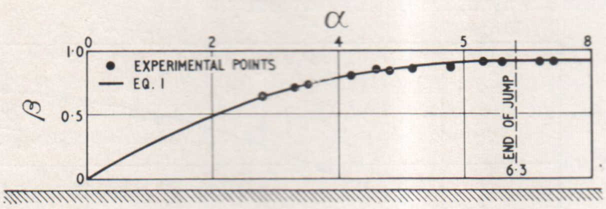

Fig. 8

an ordinate equal to $Y_1/Y_2 = 1/11{\cdot}3$ is added to the lower side and the full body of the jump is obtained as shown.

Conclusions

Based on a set of carefully conducted experiments in a one-foot-wide smooth wooden flume, it has been shown that the mean flow profile of the hydraulic jump is given by the equation

$$\beta = A_1\,\alpha + A_2\,\alpha^2$$

where A_1 and A_2 are constants, depending only on the Froude number F_1.

Acknowledgment

The material presented in this paper is taken from Ch. 6 of the thesis on "Some Studies on the Hydraulic Jump" submitted by the author for the Ph.D. degree to the Indian Institute of Science, Bangalore, in 1961. The author is thankful to Prof. N. S. Govinda Rao for his encouragement in this work.

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Vibration of Penstocks

Following an earlier article on this subject, the author describes a modified method of preventing penstock vibration which has been applied in Japan

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IN his earlier article about the vibration of penstocks in hydro-electric power plants† the author gave: (a) an approximate formula for the natural frequency of vibration of a penstock filled with water and under the influence of static head; (b) a method preventing the vibration of a penstock, together with some results of actual work of prevention. As was pointed out in that article, one way of stopping the vibration is to fix stiffener rings to the penstock, usually attached by spot-welding (Fig. 1), over a span at which vibration is taking place. This method was successfully adopted, under the author's guidance, at many hydro-electric plants in Japan.

Recently, there was a request to the author that the stiffener rings should be fixed without welding them to the penstock wall. The reasons given for this request were that the welding impaired coatings on the interior of the wall, and that it weakened the wall. In answer to this request, the author has proposed to fix stiffener rings tightly to the penstock by means of bolts. Of course, the connection must be welded after completion of the work, in order to avoid loosening, but no welding is made between the stiffener ring and the penstock wall. This method was applied successfully to vibrating penstocks of two hydro-electric power plants in Japan. One of them is at the Kowase plant, the output of which is about 8,000 kW from a vertical-shaft Francis turbine under a total head of 216 m. The diameter of the penstock is 1·50 m. The stiffener ring was made with angle-irons 130 × 130 × 9 mm, consisting of two halves which were tightened together by means of bolts (Fig. 2). The second case is Morozuka power plant, which is a pumped-storage station containing a vertical-shaft Francis turbine and a centrifugal pump in tandem and having a capacity of 50 MW under 230 m head. The diameter of the penstock is 3·20 m. In this case, the stiffener ring was made of steel plate, 200 × 16 mm in section. It consisted of three parts, and the connection was also made by bolts (Fig. 3).

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† Water Power, October 1959, p. 379.

According to the author's analytical study, the condition under which this form of stiffener ring is effective is given by the following formula;

$$2 \mid B \mid \leqq F_o \quad \dots (1)$$

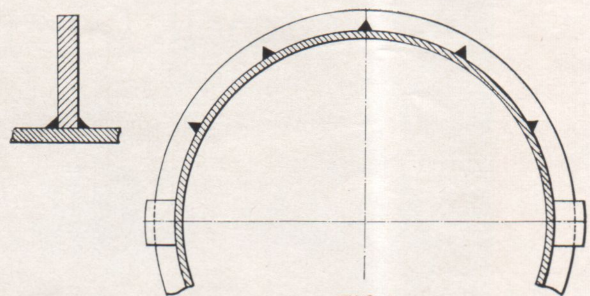

FIG.1

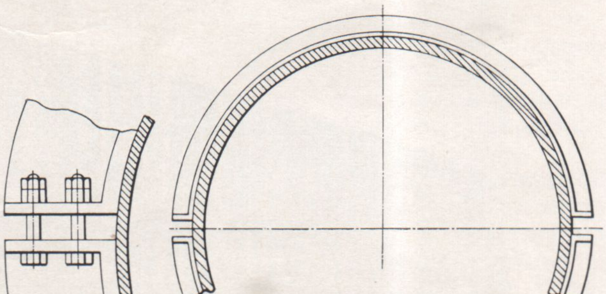

FIG.2

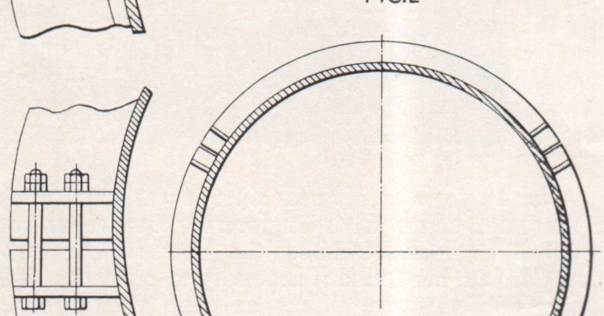

FIG.3