# A Two-Parameter Method for Determining Spillway-Apron Elevations

The article describes a two-parameter method for determining the apron elevation below ogee-profile spillways to replace methods involving trial and error or interpolations

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THE method generally used for determining apron elevations involves a trial-and-error process which is rather cumbersome and time consuming. A three-parameter method of direct solution, however, was suggested some time ago by Elevatorski[1] for high spillways, in which the loss over the spillway was neglected. A slightly modified method was suggested more recently by the author[2] for all types of spillways and taking friction loss also into account. Although these methods are a great improvement over the earlier method, they too suffer from the drawback of interpolation being involved because of their three-parameter characteristic. To overcome this difficulty the author has developed a simple two-parameter method for a given spillway profile (ogee profile has been assumed here) based on the lines indicated earlier by Rajaratnam[3]. The method is described below:

A general profile of the spillway is shown in Fig. 1 with upstream and downstream water levels, head

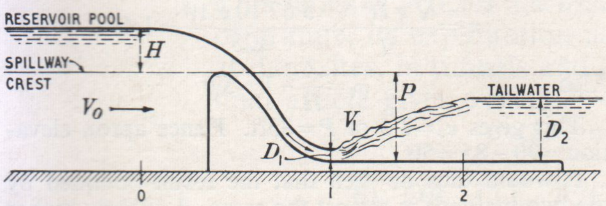


*Fig. 1*

over crest, height of spillway, depth at toe, etc., marked therein. For a high spillway ($P \geq 2H$), where $D_1$ the depth at toe is small as compared to $P+H$, the velocity of flow $V_1$ at the spillway toe, neglecting loss of head over the spillway is given by

$$V_1 = \sqrt{2g(P+H)} \qquad \ldots (1)$$

This gives:

$$q = V_1 D_1 = D_1 \sqrt{2g(P+H)} \qquad \ldots (2)$$

where $q$ is flow rate per unit width of spillway. If a coefficient $C_v$ is now introduced to take into account losses between the pool $A$ and the toe section $I$, we have

$$q = C_v D_1 \sqrt{2g(P+H)} \qquad \ldots (3)$$

Experiments carried out at Poona Research Station[4] gave $C_v = 0{\cdot}94$. However it is now found that $C_v$ is not strictly constant but depends upon $\frac{P}{H}$.

Also in terms of the head over the spillway crest, $q$ is given by:

$$q = \frac{2}{3} Cd \sqrt{2g}\, H^{3/2} = CH^{3/2} \qquad \ldots (4)$$

where $C = \frac{2}{3} Cd \sqrt{2g}$ is the overall coefficient and $Cd$ is the usual coefficient of discharge in the weir formula. Eliminating $q$ between Eqs. (3) and (4), we get:

$$\frac{D_1}{H} = \beta \frac{1}{\left(1+\frac{P}{H}\right)^{\frac{1}{2}}} \qquad \ldots (5)$$

where

$$\beta = \frac{2Cd}{3C_v} = \frac{C}{C_v \sqrt{2g}} \qquad \ldots (6)$$

If instead of coefficient $C_v$, loss $h_f$ between the pool $A$ and the toe section $I$ is considered directly, we have:

$$q = D_1 \sqrt{2g(P+H-h_f)} \qquad \ldots (7)$$

Equating (1) and (7) leads to:

$$\frac{h_f}{P+H} = 1 - C_v^2 \qquad \ldots (8)$$

There are several formulae for $h_f$, but the one that is most commonly used is Janson's formula:

$$h_f = \frac{0{\cdot}1(P+H)^{3/2}}{q^{1/3}} = \frac{0{\cdot}1(P+H)^{3/2}}{C^{1/3} H^{1/2}} \qquad \ldots (9)$$

Combining Eqs. (8) and (9), we get:

$$C_v = \sqrt{1 - \frac{0{\cdot}1(1+P/H)^{1/2}}{C^{1/3}}} \qquad \ldots (10)$$

Further for ogee shaped weirs, $C$ in the fps system of units is given by:

$$C = 3{\cdot}83 + 0{\cdot}4\frac{H}{P} \qquad \ldots (11)$$

Thus since $C$ depends on $\frac{P}{H}$, $C_v$ (Eq. 10), $\beta$ (Eq. 6) and $\frac{D_1}{H}$ (Eq. 5) also depend on $P/H$. The variation of $C_v$, $\beta$ and $\frac{D_1}{H}$ with $\frac{P}{H}$ is given in Table I.

Next, we have: $Z = P + H - D_2$, where $D_2$ is the

TABLE I

| $\frac{P}{H}$ | $C_v$ | $\beta$ | $D_1/H$ | $Z/H$ |
|---|---|---|---|---|
| 10·00 | 0·8882 | 0·5429 | 0·1715 | 8·3078 |
| 5·00 | 0·9285 | 0·524 | 0·214 | 4·0000 |
| 3·33 | 0·939 | 0·526 | 0·253 | 2·4878 |
| 2·50 | 0·944 | 0·528 | 0·282 | 1·7600 |
| 2·00 | 0·947 | 0·533 | 0·308 | 1·3270 |

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depth conjugate to depth $D_1$ at the toe
or

$$\frac{Z}{H}=\frac{P}{H}+1-\frac{D_2}{H} \quad \ldots (12)$$

also

$$D_2=\frac{D_1}{2}[-1+\sqrt{1+8F_1^2}] \quad \ldots (13)$$

where $F_1$ is Froude's Number for conditions at the spillway toe given by:

$$F_1^2=\frac{V_1^2}{gD_1}=\frac{2V_1^2}{2gD_1}=2C_v^2\left\{\frac{\frac{P}{H}+1}{D_1/H}\right\}$$

substituting this value of $F_1^2$ in Eq. (13), we have:

$$\frac{D_2}{H}=\frac{D_1}{2H}\left[-1+\sqrt{1+\frac{16C_v^2(1+P/H)}{D_1/H}}\right]$$

which coupled with Eq. (12), gives:

$$\frac{Z}{H}=\frac{P}{H}+1-\frac{D_1}{2H}\left[-1+\sqrt{1+\frac{16C_v^2(1+P/H)}{D_1/H}}\right] \quad \ldots (14)$$

Thus since $\frac{D_1}{H}$ and $C_v$ are both dependent on $P/H$, $Z/H$ will also depend on $P/H$ only. This relationship is given numerically in Table I and graphically in Fig. 2. Now given $\frac{Z}{H}$, $P/H$ and hence $P$ can be determined. Hence if we know the upstream and downstream water surface as well as the crest elevation, the apron elevation may be obtained directly.

If instead of an ogee shape, the profile corresponds to the Rehbock weir profile, the coefficient $C$ in Eq. (4) would be given by:

$$C=3{\cdot}27+0{\cdot}40\frac{H}{P} \quad \ldots (15)$$

Eq. (15) may now be used in place of Eq. (11) for computation of the values of $\beta$, $D_1/H$ and hence $Z/H$ for different values of $P/H$.

Elevatorski's[1] method gives relationship between $\pi_2$ and $\pi_3$ for different values of $\pi_1$ where, $\pi_1=\frac{\sqrt{g}H^{3/2}}{q}=\frac{\sqrt{g}}{C}$, $\pi_2=\frac{Z-H}{H}=\frac{Z}{H}-1$ and $\pi_3=\frac{P}{H}$. Thus the nature of relationship is almost similar to one given by the author here except that Elevatorski's method gives a series of curves for different values of $\pi_1$ whereas the author's method is a two-parameter method giving just a single curve. This is because for certain values of $H$ and $P$ and hence of $H/P$, $C$ and therefore $q$ is constant for a given spillway profile, whereas for different profiles, $C$ and hence $q$ vary giving different values of $\pi_1$. The same is true of the author's earlier method.

*Illustration.* Find the apron elevation required for a hydraulic-jump-type stilling basin below the ogee profile spillway portion of a dam with the following items known.

| | |
|---|---|
| Reservoir pool elevation ... ... ... | 100ft |
| Spillway crest elevation ... ... ... | 90ft |
| Tailwater elevation ... ... ... ... | 31ft |

From the given data, $\frac{Z}{H}=\frac{100-31}{100-90}=6{\cdot}9$. Hence from Fig. 2 we get, $\frac{P}{H}=8{\cdot}4$ or $P=8{\cdot}4\,H=84$ft. Hence, the apron elevation $=90-84=6$ft.

The data in this problem are just the same as given

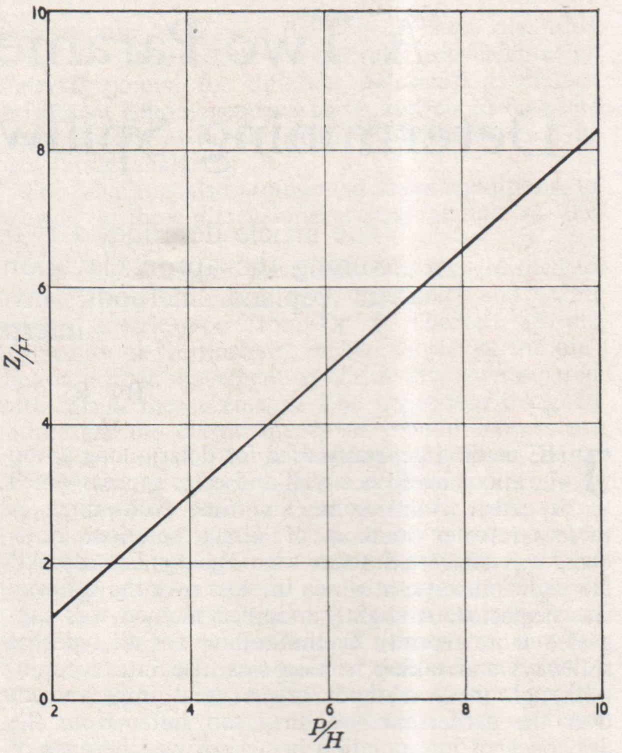


*Fig. 2*

by Elevatorski[1], except the unit discharge which is given there as 100 cusecs whereas for the ogee shaped weir assumed above it works out to 123 cusecs.

Using Elevatorski's method and taking $q=123$ cusecs we have:

$$\pi_1=\frac{\sqrt{g}H^{3/2}}{q}=\frac{5{\cdot}67\;10\sqrt{10}}{123}=1{\cdot}46$$

Also $$\pi_2=\frac{Z}{H}-1=5{\cdot}9$$

This gives $\pi_3=8{\cdot}5$ or $P=85$ft. Hence apron elevation $=90-85=5$ft.

It would thus be seen that the result obtained by the two methods is almost the same. A slightly higher value of tailwater depth obtained by Elevatorski's method could be attributed to neglect of friction loss.


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