Transactions, American Geophysical Union Volume 30, Number 6 December 1949

ANNUAL FLOODS AND THE PARTIAL-DURATION FLOOD SERIES

W. B. Langbein

Abstract--Flood data are ordinarily listed either in annual-flood series or in a partial-duration series. If the expectancy of a flood in the duration series ϵ is known, then the probability of that flood being an annual flood is shown to be $e^{-\epsilon}$. From this relationship it is possible to transform recurrence intervals in the partial duration series to those in the annual-flood series. It is shown that for equivalent floods, the recurrence intervals in the partial-duration series are smaller than in the annual-flood series, but that the difference becomes inconsequential for floods greater than about five-year recurrence interval.

There are in common use two methods of treating flood data for the study of their frequency of recurrence. The first is the annual-flood array and the second is the partial-duration series. Although most analysts take a tolerant view, some are active protagonists for one method as against the other. The differences can be shown to be largely a matter of definition.

Annual floods--An annual flood is defined as the highest peak discharge in a water year. Only the greatest flood in each year is used. An objection most frequently encountered with respect to the use of annual floods is that it uses only one flood in each year. Infrequently, the second highest flood in a given year, which the above rule omits, may outrank many annual floods.

Partial-duration series ("floods above a base")--The objection noted under annual floods is met by listing all floods above a selected base without regard to number within any given time period. The floods are numbered with respect to size, beginning with the highest as number 1. The base is generally selected as equal to the lowest annual flood so that at least one flood in each year is included. In a long record, however, the base is usually raised so that on the average only three or four floods a year are included. The only other criterion followed in the selection of the floods is that each peak be individual; that is, be separated by substantial recession in stage and discharge.

An objection to the use of the partial-flood series is that the floods listed may not be fully independent events, that is, one flood sets the stage for another. A related objection is that closely consecutive flood peaks may actually be one flood, since the damage is caused by the highest and the associated peaks may only have indirect or secondary effects on the losses.

The greater number of floods listed in the partial-duration series is commonly considered an advantage particularly when the record is short. However the greater number of points are at the low end, and the points at the upper end are generally identical with those in the annual-flood series.

A plotting of the floods for a stream by both methods, by any technique that may appeal to the analyst, will show equivalent results for the larger or less frequent floods, whereas for the smaller floods, the annual-flood graph will give results consistently below that of the partial-duration series.

Consider the one-year flood. For the annual-flood series, this is represented by the lowest of the annual floods. For the partial-duration series, this is commonly taken as the Nth flood in an N-year record. Obviously, the first is less than the second.

In the first case the one-year flood is that which may be equaled or exceeded in any year. In the second case it is that which has a mean expectancy, ϵ, of one in a year.

The probability of recurrence of a flood equal to or greater than magnitude m is in general $P = m/nN$, where n is the average number of floods per year, N is the total number of years of record, and m is the order of magnitude beginning with the highest. In the annual-flood series, $n = 1$ hence $p = m/N$. In the partial-duration series it is not necessary to define n, since the equation $pn = \epsilon$, defines the annual expectancy of recurrence of a flood that is equal to or greater than magnitude m. Hence in this case $m/N = \epsilon$.

If there are n flood peaks a year on the average, distributed randomly among N years, consider

Table 1--Relationship between flood expectancies in the partial-duration series and the probability of the corresponding flood as an annual flood

Expectancies partial-duration series m/N	Probabilities as annual floods	
	$p=e^{-m/N}$	$1-p = 1-e^{-m/N}$
2.0	0.135	0.865
1.0	0.368	0.632
0.69	0.500	0.500
0.50	0.606	0.394
0.20	0.819	0.181
0.10	0.905	0.095
0.05	0.951	0.049
0.02	0.9802	0.0198
0.01	0.99005	0.00995

Table 2--Relationship of recurrence interval of floods on the two bases

Partial-duration series	Annual floods
yr	yr
0.5	1.16
1.0	1.58
1.45	2.00
2.0	2.54
5.0	5.52
10	10.5
20	20.5
50	50.5
100	100.5

a flood of average expectancy $m/N = \epsilon$ (that is, expectancy that the flood will be of magnitude m or greater in a year). Then the probability of any flood being of this magnitude or greater is ϵ/n, and the complementary probability of a flood equal to or less than magnitude m is $1 - \epsilon/n$. Accordingly the probability of a flood of magnitude m being a maximum of the n floods in a year will be $p = (1 - \epsilon/n)^n = e^{-\epsilon}$, provided ϵ is small compared with n. The value of p therefore represents the probability of a flood of magnitude m, being an annual flood, and ϵ represents its expectancy m/N, among all floods in the partial-duration series. The value of 1 - p equals the probability of a flood of magnitude, m, being equalled or exceeded in the annual-flood series.

Table 1 may be constructed to show the relationship between flood expectancies in the partial-duration series and the probability of the corresponding flood as an annual flood and may be used to plot a partial-duration series on probability graphs. The third column gives the computed probabilities corresponding to observed expectancies listed in the first column. From this table we may also

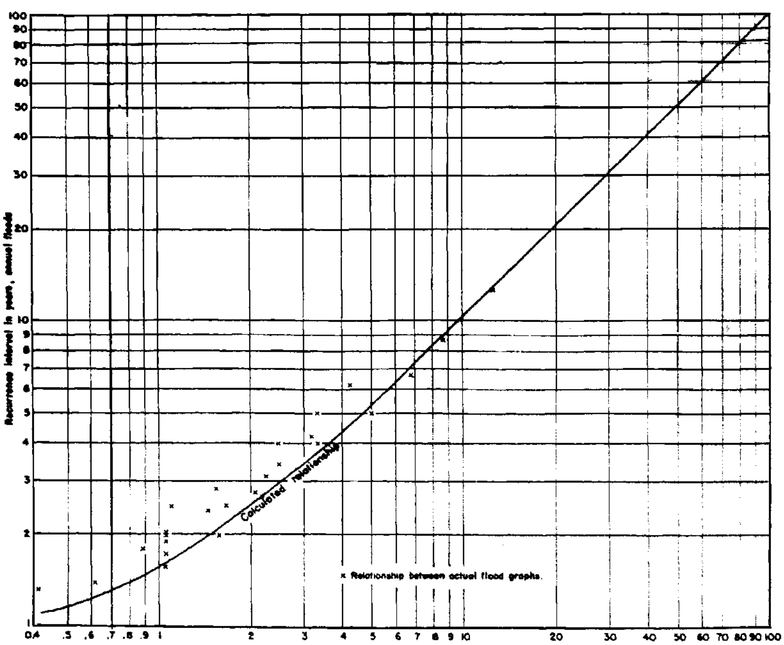

Fig. 1--Study of relationship between recurrence interval, partial-duration series and annual floods

construct Table 2 to show the relationship between recurrence intervals of floods in the partial-duration series and as annual floods. The items in Table 2 are the reciprocals of corresponding items in the first and third columns of Table 1.

It will be noted that the two scales differ markedly for the smaller or more frequent floods, but are nearly equal for the higher floods. An annual flood is the maximum of all floods in a given year whereas a flood in the partial-duration series is selected as exceeding an arbitrary base and without reference to the number of other floods in the year. However, since a large flood is apt to outrank any other flood in the year in which it occurs, the recurrence intervals of great floods are closely the same in both scales.

The Tables were computed on the postulate that floods occur as completely independent events. Floods rarely behave according to the strict definition of randomness implied. Hydrologists recognize a persistence or interdependence in floods. Accordingly, there are deviations from random distribution, chiefly in the direction of greater grouping of the larger discharges, so as to make the items in the first column somewhat low.

Figure 1 shows the relationship indicated by Table 2. Also shown are a few points taken by comparing partial-duration series with annual floods for a few actual cases. The plotted points tend to plot above their calculated positions.

There is an important distinction in meaning as between the recurrence interval of these floods. In the annual-flood series the recurrence interval is the average interval in which a flood of given size will recur as an annual maximum. In the partial-duration series, the recurrence interval is the average interval between floods of a given size regardless of their relationship to the year or any other period of time. This distinction remains, even though for large floods the two approach numerical equality.

U. S. Geological Survey,
Washington, D. C.

(Communicated manuscript received December 23, 1948;
open for formal discussion until May 1, 1950.)