



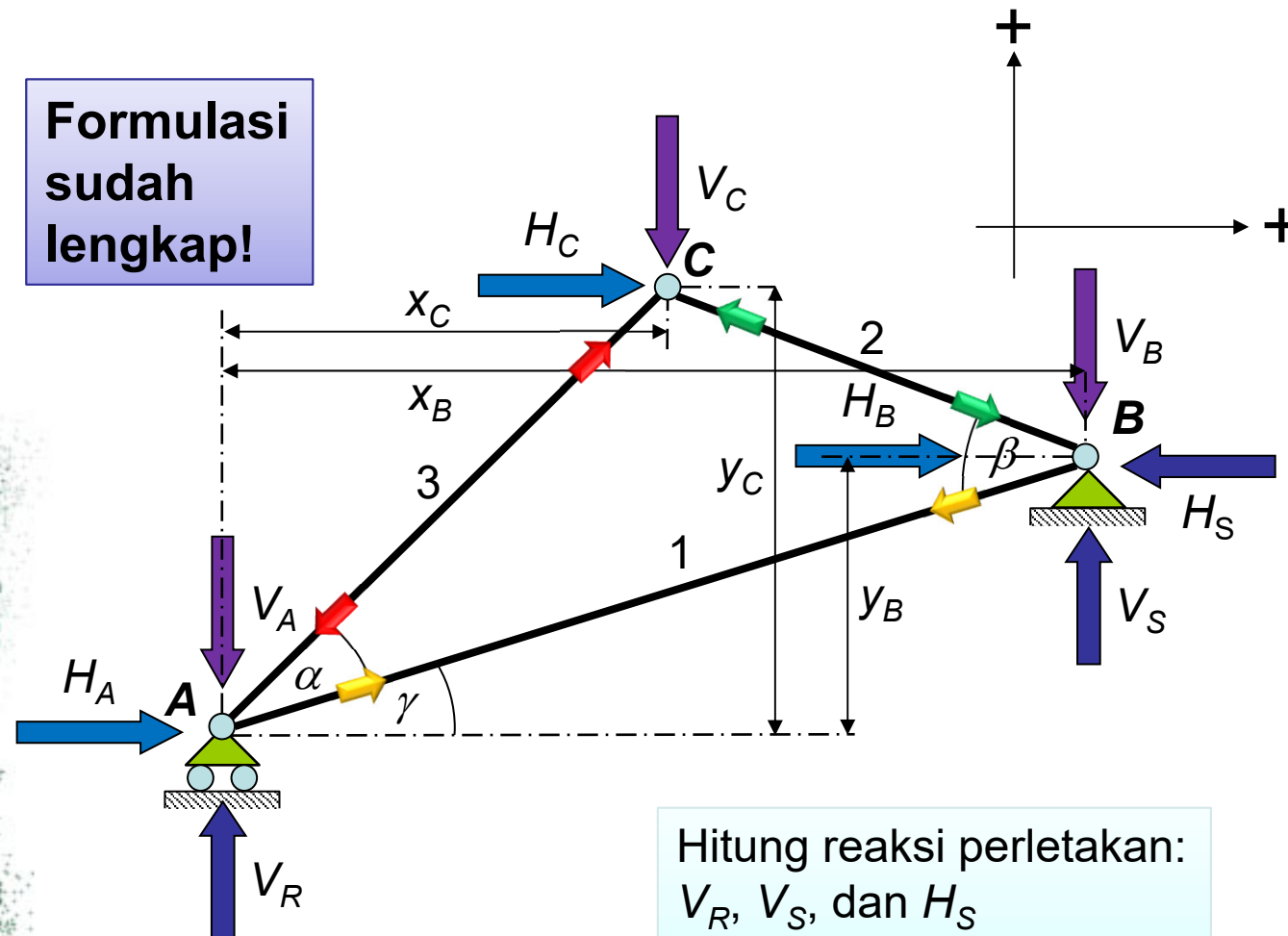
Metoda Numerik

Aplikasi di Lapangan: Kuda-kuda Asimetri

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Kuda-kuda Asimetri

Formulasi
sudah
lengkap!



Hitung reaksi perletakan:
 V_R , V_S , dan H_S

Hitung gaya-gaya internal yaitu gaya-gaya batang:
 P_1 , P_2 , dan P_3



Hitungan Reaksi Perletakan dan Gaya Batang

- Diketahui:

$$x_B, x_C, y_B, y_C, V_A, H_A, V_B, H_B, V_C, H_C$$

- Dihitung:

$$V_R, V_S, H_S, P_1, P_2, P_3$$

- Reaksi perletakan dihitung dengan cara:

- $\Sigma F_H = 0$
- $\Sigma F_V = 0$
- $\Sigma M_B = 0$

Hitungan Reaksi Perletakan

Reaksi perletakan dihitung dengan cara:

$$1. \Sigma F_H = 0 \rightarrow H_A + H_B + H_C + H_s = 0$$
$$H_s = -H_A - H_B - H_C$$

$$2. \Sigma F_V = 0 \rightarrow V_R + V_S + V_A + V_B + V_C = 0$$
$$V_R + V_S = -V_A - V_B - V_C$$

$$3. \Sigma M_B = 0 \rightarrow (V_R + V_A) x_B - H_A y_B + H_C (y_C - y_B) + V_C (x_B - x_C) = 0$$
$$V_R x_B = H_A y_B - H_C (y_C - y_B) - V_C (x_B - x_C) - V_A y_B$$

Kesetimbangan Titik buhul A

4. $\Sigma F_H = 0$

- $P_1 \cos \gamma - P_3 \cos (\alpha + \gamma) + H_A = 0$
- $-P_1 \cos \gamma + P_3 \cos (\alpha + \gamma) = H_A$

5. $\Sigma F_V = 0$

- $V_R + V_A + P_1 \sin \gamma - P_3 \sin (\alpha + \gamma) = 0$
- $-P_1 \sin \gamma + P_3 \sin (\alpha + \gamma) = V_A + V_R$

Kesetimbangan Titik buhul *B*

$$6. \Sigma F_H = 0$$

- $-P_1 \cos \gamma + P_2 \cos (\beta - \gamma) + H_B + H_S = 0$
- $P_1 \cos \gamma - P_2 \cos (\beta - \gamma) = H_B + H_S$

$$7. \Sigma F_V = 0$$

- $V_B + V_S - P_1 \sin \gamma - P_2 \sin (\beta - \gamma) = 0$
- $P_1 \sin \gamma + P_2 \sin (\beta - \gamma) = V_B + V_S$

Kesetimbangan Titik buhul C

8. $\Sigma F_H = 0$

- $-P_2 \cos (\beta - \gamma) + P_3 \cos (\alpha + \gamma) + H_C = 0$
- $P_2 \cos (\beta - \gamma) - P_3 \cos (\alpha + \gamma) = H_C$

9. $\Sigma F_V = 0$

- $P_2 \sin (\beta - \gamma) + P_3 \sin (\alpha + \gamma) + V_C = 0$
- $-P_2 \sin (\beta - \gamma) - P_3 \sin (\alpha + \gamma) = V_C$

Sistem persamaan linier

Dipilih 6 dari 9 persamaan yang dibutuhkan untuk menghitung: V_R , V_S , H_S , P_1 , P_2 , P_3

1. $V_R = \{H_A y_B - H_C (y_C - y_B) - V_C (x_B - x_C) - V_A x_B\} / x_B$
2. $V_R + V_S = -V_A - V_B - V_C$
3. $H_S = -H_A - H_B - H_C$
4. $P_1 \cos \gamma - P_2 \cos (\beta - \gamma) = H_B + H_S$
5. $P_2 \cos (\beta - \gamma) - P_3 \cos (\alpha + \gamma) = H_C$
6. $-P_2 \sin (\beta - \gamma) - P_3 \sin (\alpha + \gamma) = V_C$

Modifikasi sistem persamaan linier

1. $V_R = \{H_A y_B - H_C (y_C - y_B) - V_C (x_B - x_C) - V_A x_B\} / x_B$
2. $V_R + V_S = -V_A - V_B - V_C$
3. $H_S = -H_A - H_B - H_C$
4. $-P_1 \cos \gamma - P_2 \cos (\beta - \gamma) = H_B + H_S$
5. $-P_2 \cos (\beta - \gamma) + P_3 \cos (\alpha + \gamma) = H_C$
6. $-P_2 \sin (\beta - \gamma) - P_3 \sin (\alpha + \gamma) = V_C$

Reaksi perletakan dihitung terlebih dahulu:

1. $V_R = \{H_A y_B - H_C (y_C - y_B) + V_C x_C\} / x_B - V_A - V_C$
2. $V_S = -V_A - V_B - V_C - V_R$
 $V_S = -\{H_A y_B - H_C (y_C - y_B) + V_C x_C\} / x_B - V_B$
3. $H_S = -H_A - H_B - H_C$

Sistem persamaan linier final

1. $V_R = \{H_A y_B - H_C (y_C - y_B) + V_C x_C\} / x_B - V_A - V_C$
2. $V_S = -\{H_A y_B - H_C (y_C - y_B) + V_C x_C\} / x_B - V_B$
3. $H_s = -H_A - H_B - H_C$
4. $P_1 \cos \gamma - P_2 \cos (\beta - \gamma) = H_B + H_S$
5. $P_2 \cos (\beta - \gamma) - P_3 \cos (\alpha + \gamma) = H_C$
6. $-P_2 \sin (\beta - \gamma) - P_3 \sin (\alpha + \gamma) = V_C$

Persamaan 4, 5, dan 6 diselesaikan untuk menghitung: P_1, P_2, P_3

$$\begin{bmatrix} \cos \gamma & -\cos(\beta - \gamma) & 0 \\ 0 & \cos(\beta - \gamma) & -\cos(\alpha + \gamma) \\ 0 & -\sin(\beta - \gamma) & -\sin(\alpha + \gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$

Rekapitulasi Hitungan

- Reaksi perletakan

$$V_R = \frac{1}{x_B} \{ H_A y_B - H_C (y_C - y_B) + V_C x_C \} - V_A - V_C$$

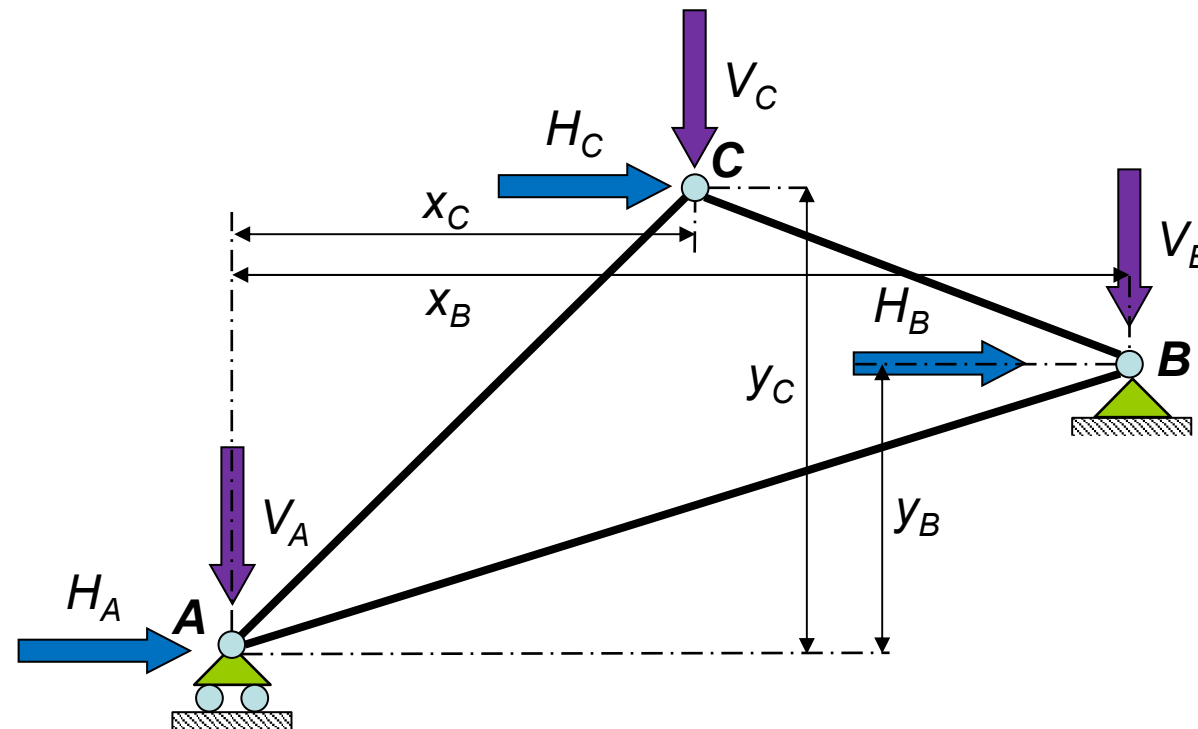
$$V_S = \frac{-1}{x_B} \{ H_A y_B - H_C (y_C - y_B) + V_C x_C \} - V_B$$

$$H_S = -H_A - H_B - H_C$$

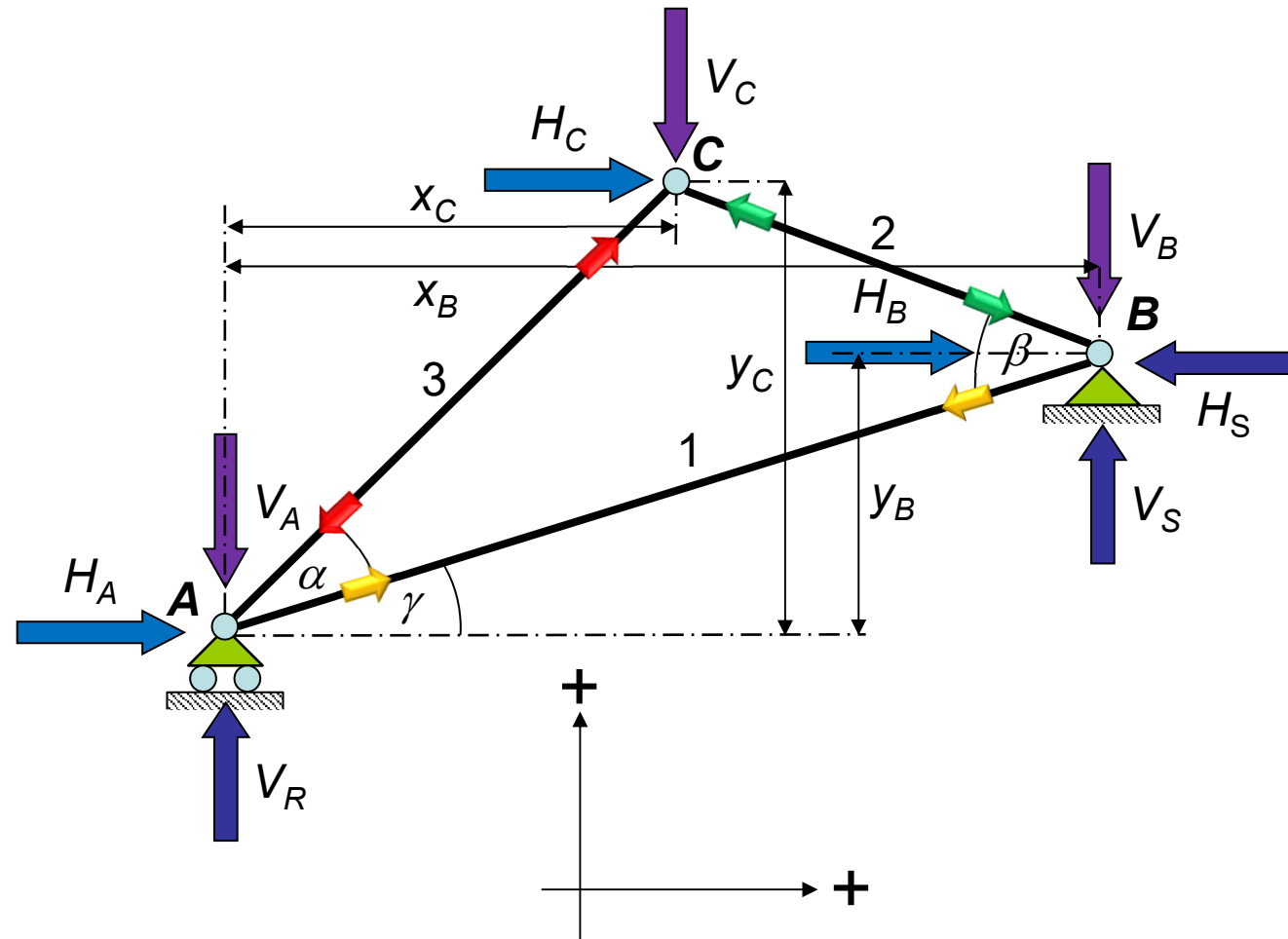
- Gaya batang

$$\begin{bmatrix} \cos \gamma & -\cos(\beta - \gamma) & 0 \\ 0 & \cos(\beta - \gamma) & -\cos(\alpha + \gamma) \\ 0 & -\sin(\beta - \gamma) & -\sin(\alpha + \gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$

Gambar Soal



Gambar Jawaban



Eliminasi Gauss 1/2

$$\begin{bmatrix} \cos \gamma & -\cos(\beta - \gamma) & 0 \\ 0 & \cos(\beta - \gamma) & -\cos(\alpha + \gamma) \\ 0 & -\sin(\beta - \gamma) & -\sin(\alpha + \gamma) \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} H_B + H_S \\ H_C \\ V_C \end{Bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{\cos(\beta - \gamma)}{\cos \gamma} & 0 \\ 0 & 1 & \frac{-\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} \\ 0 & 1 & \frac{\sin(\alpha + \gamma)}{\sin(\beta - \gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \frac{H_B + H_S}{\cos \gamma} \\ \frac{H_C}{\cos(\beta - \gamma)} \\ \frac{V_C}{-\sin(\beta - \gamma)} \end{Bmatrix}$$

Eliminasi Gauss 2/2

$$\begin{bmatrix} 1 & -\frac{\cos(\beta - \gamma)}{\cos \gamma} & 0 \\ 0 & 1 & \frac{-\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} \\ 0 & 1 & \frac{\sin(\alpha + \gamma)}{\sin(\beta - \gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \frac{H_B + H_S}{\cos \gamma} \\ \frac{H_C}{\cos(\beta - \gamma)} \\ \frac{V_C}{-\sin(\beta - \gamma)} \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -\frac{\cos(\beta - \gamma)}{\cos \gamma} \frac{\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} \\ 0 & 1 & \frac{-\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} \\ 0 & 0 & \frac{\sin(\alpha + \gamma)}{\sin(\beta - \gamma)} + \frac{\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} \frac{H_B + H_C + H_S}{\cos \gamma} \\ \frac{H_C}{\cos(\beta - \gamma)} \\ \frac{V_C}{-\sin(\beta - \gamma)} - \frac{H_C}{\cos(\beta - \gamma)} \end{Bmatrix}$$

Substitusi

$$\begin{aligned} P_3 &= \frac{\frac{V_C}{-\sin(\beta - \gamma)} - \frac{H_C}{\cos(\beta - \gamma)}}{\frac{\sin(\alpha + \gamma)}{\sin(\beta - \gamma)} + \frac{\cos(\alpha + \gamma)}{\cos(\beta - \gamma)}} \\ &= -\frac{V_C + H_C \tan(\beta - \gamma)}{\sin(\alpha + \gamma) + \tan(\beta - \gamma) \cos(\alpha + \gamma)} \end{aligned}$$

$$P_2 = \frac{H_C}{\cos(\beta - \gamma)} + \frac{\cos(\alpha + \gamma)}{\cos(\beta - \gamma)} P_3$$

$$P_1 = \frac{H_B + H_C + H_S}{\cos \gamma} + \frac{\cos(\alpha + \gamma)}{\cos \gamma} P_3$$